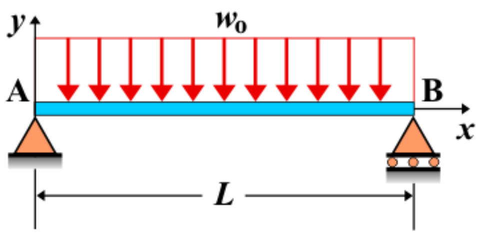
| Symbol | Physical quantity | Units |
|---|---|---|
| *E·I* | Flexural rigidity | $N·m^2$, $Pa·m^4$ |
| *y* | Deflection or deformation | m |
| $\theta$ | Slope, Slope of the deflection, Angle of rotation | - |
| *x* | Distance from support (origin) | m |
| *L* | Length of beam (without overhang) | m |
| *M* | Moment, Bending moment, Couple moment applied | N·m |
| *P* | Concentrated load, Point load, Concentrated force | N |
| *w* | Distributed load, Load per unit length | N/m |
| *R* | Reaction load, reaction force | N |
| *V* | Shear force, shear | N |

## Simple beam - Uniformly distributed load




Deflection $y_{AB} = \frac{-w_0 x}{24EI}(L^3 - 2Lx^2 + x^3)$

$y_{MAX} = \frac{-5w_0 L^4}{384EI}$ at $x = \frac{L}{2}$

Slope $\theta_{AB} = \frac{-w_0}{24EI}(L^3 - 6Lx^2 + 4x^3)$

$\theta_A = -\theta_B = \frac{-w_0 L^3}{24EI}$

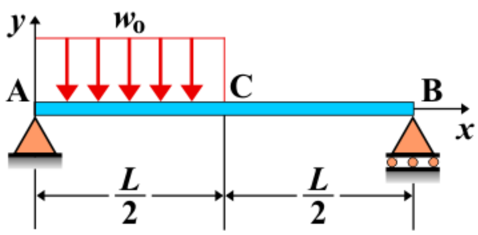
Moment $M_{AB} = \frac{w_0 x}{2}(L - x)$

$M_{MAX} = \frac{w_0 L^2}{8}$ at $x = \frac{L}{2}$

Shear $V_{AB} = \frac{w_0}{2}(L - 2x)$

Reactions $R_A = R_B = \frac{w_0 L}{2}$

**Simple beam - Uniform load partially distributed at left end (I)**




Deflection $y_{AC} = \frac{-w_0 x}{384EI}(9L^3 - 24Lx^2 + 16x^3)$

$$y_{CB} = \frac{-w_0 L}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$$

Slope $\theta_{AC} = \frac{-w_0}{384EI}(9L^3 - 72Lx^2 + 64x^3)$

$$\theta_{CB} = \frac{-w_0 L}{384EI}(24x^2 - 48Lx + 17L^2)$$

$$\theta_A = \frac{-3wL^3}{128EI} \qquad \theta_B = \frac{7wL^3}{384EI}$$

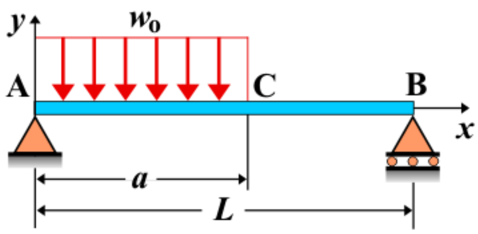
Moment $M_{AC} = \frac{w_0}{8}(3Lx - 4x^2) \qquad M_{CB} = \frac{w_0}{8}(L^2 - Lx)$

Shear $V_{AC} = \frac{w_0}{8}(3L - 8x) \qquad V_{CB} = \frac{-w_0 L}{8}$

$$V_A = R_A \qquad V_B = -R_B$$

Reactions $R_A = \frac{3w_0 L}{8} \qquad R_B = \frac{w_0 L}{8}$

**Simple beam - Uniform load partially distributed at left end (II)**




Deflection:

$$y_{AC} = \frac{-w_0 x}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 2a^2x^2 - 4aLx^2 + Lx^3)$$

$$y_{CB} = \frac{-w_0 a^2}{24LEI}(-a^2L + 4L^2x + a^2x - 6Lx^2 + 2x^3)$$

Slope:

$$\theta_{AC} = \frac{-w_0}{24LEI}(a^4 - 4a^3L + 4a^2L^2 + 6a^2x^2 - 12aLx^2 + 4Lx^3)$$

$$\theta_{CB} = \frac{-w_0 a^2}{24LEI}(4L^2 + a^2 - 12Lx + 6x^2)$$

Moment:

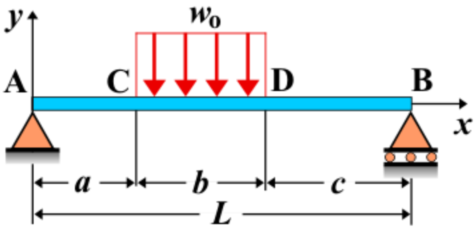
$$M_{AC} = \frac{-w_0}{2L}(a^2x - 2aLx + Lx^2) \qquad M_{CB} = \frac{w_0 a^2}{2L}(L - x)$$

Shear:

$$V_{AC} = \frac{-w_0}{2L}(a^2 - 2aL + 2Lx) \qquad V_{CB} = V_C = V_B = \frac{-w_0 a^2}{2L}$$

Reactions $R_A = \frac{w_0 a}{2L}(2L - a) \qquad R_B = \frac{w_0 a^2}{2L}$

## Simple beam - Uniform load partially distributed




Deflection $y_{AC} = \dfrac{R_A x^3}{6EI} + \alpha x \qquad y_{CD} = \dfrac{R_A x^3}{6EI} - \dfrac{w_0}{24EI}(x-a)^4 + \alpha x$

$$y_{DB} = \frac{R_B (L-x)^3}{6EI} + \frac{\beta (L-x)}{L}$$

Slope: $\theta_{AC} = \dfrac{R_A x^2}{2EI} + \alpha \qquad \theta_{CD} = \dfrac{R_A x^2}{2EI} - \dfrac{w_0}{6EI}(x-a)^3 + \alpha$

$$\theta_{DB} = \frac{-R_B (L-x)^2}{2EI} - \frac{\beta}{L}$$

Moment $M_{AC} = R_A x \qquad M_{CD} = R_A x - \dfrac{w_0}{2}(x-a)^2$

$$M_{DB} = R_B (L-x)$$

Shear $V_{AC} = V_A = V_C = R_A \qquad V_{CD} = R_A - w_0 (x-a)$

$$V_{DB} = V_D = V_B = -R_B$$

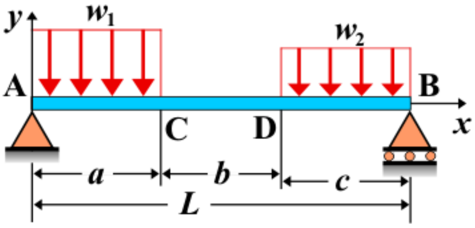
Reactions $R_A = \dfrac{w_0 b}{2L}(2c+b) \qquad R_B = \dfrac{w_0 b}{2L}(2a+b)$

Where:

$$\alpha = \frac{w_0 b^3 L - 6EI\beta - 3R_B c^2 L - 3R_A L(a+b)^2}{6LEI}$$

$$\beta = \frac{4w_0 ab^3 + 3w_0 b^4 - 8R_A (a+b)^3 - 12R_B c^2 L + 8R_B c^3}{24EI}$$

## Simple beam - Uniform load partially distributed at each end




Moment $M_{AC} = R_A x - \dfrac{w_1 x^2}{2}$

$$M_{CD} = R_A x - \frac{w_1 a}{2}(2x-a) \qquad M_{DB} = R_B (L-x) - \frac{w_2 (L-x)^2}{2}$$

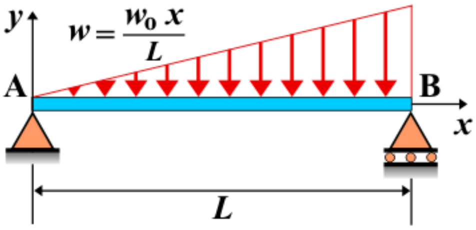
Shear:

$$V_{AC} = R_A - w_1 x \qquad V_{CD} = R_A - w_1 a \qquad V_{DB} = -R_B + w_2 (L-x)$$

Reactions:

$$R_A = \frac{w_1 a(2L-a) + w_2 c^2}{2L} \qquad R_B = \frac{w_2 c(2L-c) + w_1 a^2}{2L}$$

**Simple beam - Load increasing uniformly to right end**




Deflection $$y_{AB} = \frac{-w_0 x}{360LEI}(7L^4 - 10L^2x^2 + 3x^4)$$

$$y_{MAX} = -0{,}00652\frac{w_0 L^4}{EI} \quad \text{at} \quad x = 0{,}5193\text{L}$$

Slope $$\theta_{AB} = \frac{-w_0}{360LEI}(7L^4 - 30L^2x^2 + 15x^4)$$

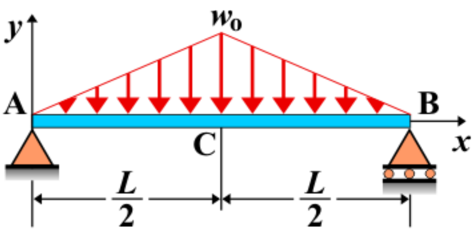
$$\theta_A = \frac{-7w_0 L^3}{360EI} \qquad \theta_B = \frac{w_0 L^3}{45EI}$$

Moment $$M_{AB} = \frac{w_0}{6L}(L^2 x - x^3)$$

Shear $$V_{AB} = \frac{w_0}{6L}(L^2 - 3x^2)$$

Reactions $$R_A = \frac{w_0 L}{6} \qquad R_B = \frac{2w_0 L}{6}$$

**Simple beam - Load increasing uniformly to center**




Deflection $$y_{AC} = \frac{-w_0 x}{960LEI}(5L^2 - 4x^2)^2$$

$$y_{CB} = \frac{-w_0(L-x)}{960LEI}(5L^2 - 4(L-x)^2)^2$$

$$y_{MAX} = \frac{-w_0 L^4}{120EI} \quad \text{at} \quad x = \frac{L}{2}$$

Slope $$\theta_{AC} = \frac{-w_0}{192LEI}(5L^2 - 4x^2)(L^2 - 4x^2)$$

$$\theta_{CB} = \frac{w_0}{192LEI}(5L^2 - 4(L-x)^2)(L^2 - 4(L-x)^2)$$

$$\theta_A = -\theta_B = \frac{-5w_0 L^3}{192EI}$$

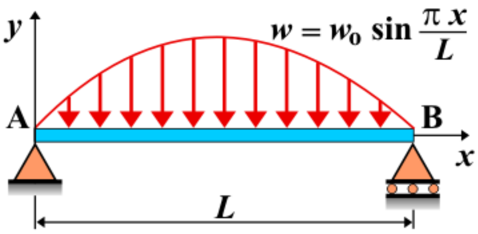
Moment $$M_{AC} = \frac{w_0}{12L}(3L^2 x - 4x^3)$$

$$M_{CB} = \frac{w_0(L-x)}{12L}(3L^2 - 4(L-x)^2)$$

Shear $$V_{AC} = \frac{w_0}{4L}(L^2 - 4x^2) \qquad V_{CB} = \frac{-w_0}{4L}(L^2 - 4(L-x)^2)$$

Reactions $$R_A = R_B = \frac{w_0 L}{4}$$

### Simple beam - Sinusoidal distributed load




Deflection $y_{AB} = \frac{-w_0 L^4}{\pi^4 EI} \sin \frac{\pi x}{L}$

$y_{MAX} = \frac{-w_0 L^4}{\pi^4 EI}$ at $x = \frac{L}{2}$

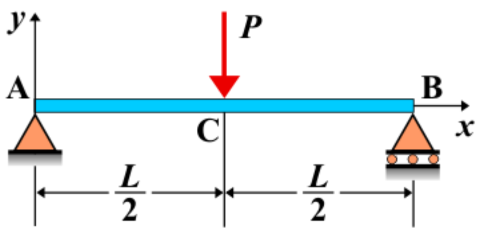
Slope $\theta_{AB} = \frac{-w_0 L^3}{\pi^3 EI} \cos \frac{\pi x}{L}$ $\theta_A = -\theta_B = \frac{-w_0 L^3}{\pi^3 EI}$

Moment $M_{AB} = \frac{w_0 L^2}{\pi^2} \sin \frac{\pi x}{L}$

Shear $V_{AB} = \frac{w_0 L}{\pi} \cos \frac{\pi x}{L}$ $V_A = -V_B = \frac{w_0 L}{\pi}$

Reactions $R_A = R_B = \frac{w_0 L}{\pi}$

### Simple beam - Concentrated load at center




Deflection $y_{AC} = \frac{-Px}{48EI}(3L^2 - 4x^2)$

$y_{CB} = \frac{-P(L-x)}{48EI}(3L^2 - 4(L-x)^2)$

$y_{MAX} = y_C = \frac{-PL^3}{48EI}$ at $x = \frac{L}{2}$

Slope:

$\theta_{AC} = \frac{-P}{16EI}(L^2 - 4x^2)$ $\theta_{CB} = \frac{-P}{16EI}(4x^2 - 8Lx + 3L^2)$

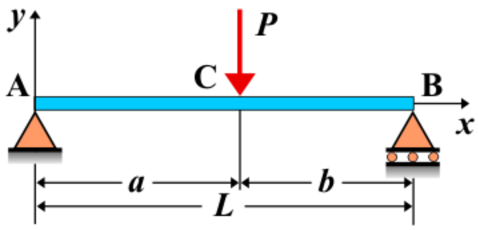
$\theta_A = -\theta_B = \frac{PL^2}{16EI}$

Moment $M_{AC} = \frac{Px}{2}$ $M_{CB} = \frac{P(L-x)}{2}$

Shear $V_{AC} = V_A = \frac{P}{2}$ $V_{CB} = V_B = \frac{-P}{2}$

Reactions $R_A = R_B = \frac{P}{2}$

### Simple beam - Concentrated load at any point




Deflection $y_{AC} = \frac{-Pbx}{6LEI}(L^2 - b^2 - x^2)$

$$y_{CB} = \frac{-Pa(L-x)}{6LEI}\left[L^2 - a^2 - (L-x)^2\right]$$

Slope:

$$\theta_{AC} = \frac{-Pb}{6LEI}(L^2 - b^2 - 3x^2) \quad \theta_{CB} = \frac{Pa}{6LEI}\left[L^2 - a^2 - 3(L-x)^2\right]$$

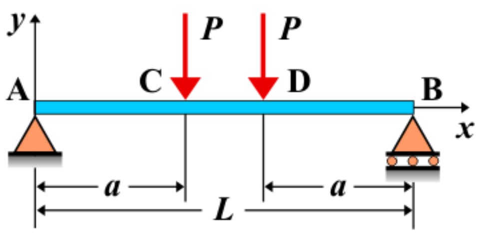
$$\theta_A = \frac{-Pb(L^2 - b^2)}{6LEI} \quad \theta_B = \frac{Pa}{6LEI}(L^2 - a^2)$$

Moment $M_{AC} = \frac{Pbx}{L}$ $\quad M_{CB} = \frac{Pa(L-x)}{L}$

Shear $V_{AC} = V_A = \frac{Pb}{L}$ $\quad V_{CB} = V_B = \frac{-Pa}{L}$

Reactions $R_A = \frac{Pb}{L}$ $\quad R_B = \frac{Pa}{L}$

### Simple beam - Two equal concentrated loads symmetrically placed




Deflection $y_{AC} = \frac{-Px}{6EI}(3aL - 3a^2 - x^2)$

$$y_{CD} = \frac{-Pa}{6EI}(3Lx - 3x^2 - a^2)$$

$$y_{DB} = \frac{-P(L-x)}{6EI}\left[3aL - 3a^2 - (L-x)^2\right]$$

$$y_{MAX} = \frac{-Pa}{24EI}(3L^2 - 4a^2) \text{ at } x = \frac{L}{2}$$

Slope $\theta_{AC} = \frac{-P}{2EI}(aL - a^2 - x^2)$ $\quad \theta_{CD} = \frac{-Pa}{2EI}(L - 2x)$

$$\theta_{DB} = \frac{P}{2EI}\left[aL - a^2 - (L-x)^2\right]$$

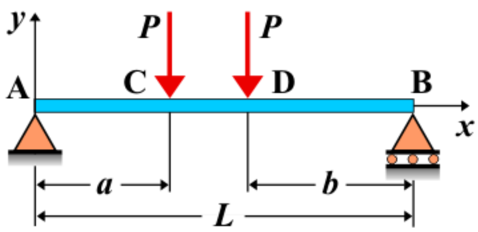
$$\theta_A = -\theta_B = \frac{-P(aL - a^2)}{2EI}$$

Moment $M_{AC} = Px$ $\quad M_{CD} = Pa$ $\quad M_{DB} = P(L-x)$

Shear $V_{AC} = P$ $\quad V_{CD} = 0$ $\quad V_{DB} = -P$

Reactions $R_A = R_B = P$

### Simple beam - Two equal concentrated loads unsymmetrically placed



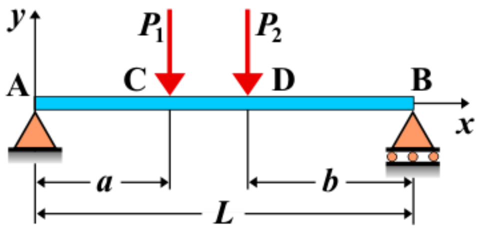

Moment $M_{AC} = R_A x$ $\quad M_{CD} = R_A x - P(x-a)$

$$M_{DB} = R_B(L-x)$$

Shear $V_{AC} = R_A$ $\quad V_{CD} = R_A - P$ $\quad V_{DB} = -R_B$

Reactions $R_A = \frac{P(L-a+b)}{L}$ $\quad R_B = \frac{P(L-b+a)}{L}$

### Simple beam - Two unequal concentrated loads unsymmetrically placed



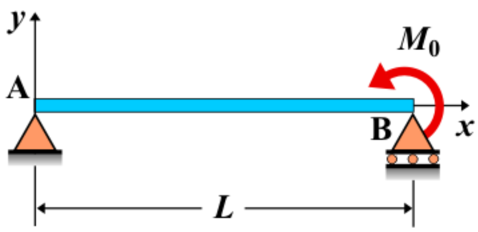

Moment $M_{AC} = R_A x \qquad M_{CD} = R_A x - P_1(x-a)$

$M_{DB} = R_B(L-x)$

Shear $V_{AC} = R_A \qquad V_{CD} = R_A - P_1 \qquad V_{DB} = -R_B$

Reactions $R_A = \dfrac{P_1(L-a)+P_2 b}{L} \qquad R_B = \dfrac{P_2(L-b)+P_1 a}{L}$

### Simple beam - Couple moment Mo at right end




Deflection $y_{AB} = \dfrac{-M_0 x}{6LEI}(L^2 - x^2)$

Slope $\theta_{AB} = \dfrac{-M_0}{6LEI}(L^2 - 3x^2)$

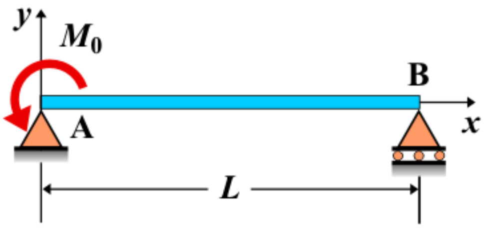
$\theta_A = \dfrac{-M_0 L}{6EI} \qquad \theta_B = \dfrac{M_0 L}{3EI}$

Moment $M_{AB} = \dfrac{M_0 x}{L}$

Shear $V_{AB} = \dfrac{M_0}{L}$

Reactions $R_A = \dfrac{M_0}{L} \qquad R_B = \dfrac{-M_0}{L}$

### Simple beam - Couple moment Mo at left end (I)




Deflection $y_{AB} = \dfrac{M_0 x}{6LEI}(2L^2 - 3Lx + x^2)$

$y_{MAX} = \dfrac{M_0 L^2}{9\sqrt{3}EI}$ at $x = \left(\dfrac{3-\sqrt{3}}{3}\right)L$

Slope $\theta_{AB} = \dfrac{M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$

$\theta_A = \dfrac{M_0 L}{3EI} \qquad \theta_B = \dfrac{-M_0 L}{6EI}$

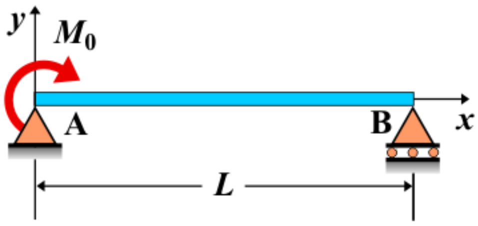
Moment $M_{AB} = \dfrac{-M_0}{L}(L-x)$

Shear $V_{AB} = \dfrac{M_0}{L}$

Reactions $R_A = \dfrac{M_0}{L} \qquad R_B = \dfrac{-M_0}{L}$

**Simple beam - Couple moment Mo at left end (II)**

<table>
<tr><td>




</td><td>

Deflection $y_{AB} = \frac{-M_0 x}{6LEI}(2L^2 - 3Lx + x^2)$

$y_{MAX} = \frac{-M_0 L^2}{9\sqrt{3}EI}$ at $x = \left(\frac{3-\sqrt{3}}{3}\right)L$

Slope $\theta_{AB} = \frac{-M_0}{6LEI}(2L^2 - 6Lx + 3x^2)$

$\theta_A = \frac{-M_0 L}{3EI}$ $\theta_B = \frac{M_0 L}{6EI}$

Moment $M_{AB} = \frac{M_0}{L}(L - x)$

Shear $V_{AB} = \frac{-M_0}{L}$

Reactions $R_A = \frac{-M_0}{L}$ $R_B = \frac{M_0}{L}$

</td></tr>
</table>

**Simple beam - Couple moment Mo at center**

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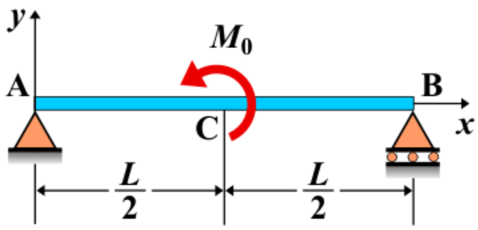
<tr><td>

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Deflection $y_{AC} = \frac{-M_0 x}{24LEI}(L^2 - 4x^2)$

$y_{CB} = \frac{M_0 (L-x)}{24LEI}(L^2 - 4(L-x)^2)$

Slope $\theta_{AC} = \frac{-M_0}{24LEI}(L^2 - 12x^2)$

$\theta_{CB} = \frac{M_0}{24LEI}(12(L-x)^2 - L^2)$

$\theta_A = \frac{-M_0}{6LEI}(L^2 - 3b^2)$ $\theta_B = \frac{M_0}{6LEI}(-L^2 + 3a^2)$

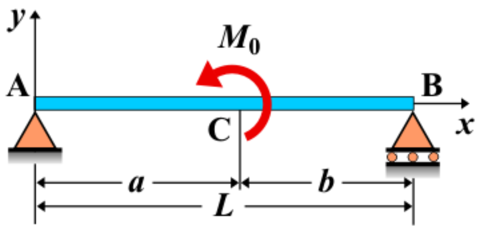
Moment $M_{AC} = \frac{M_0 x}{L}$ $M_{CB} = \frac{-M_0}{L}(L - x)$

Shear $V_{AC} = \frac{M_0}{L}$ $V_{CB} = \frac{M_0}{L}$

Reactions $R_A = \frac{M_0}{L}$ $R_B = \frac{-M_0}{L}$

</td></tr>
</table>

## Simple beam - Couple moment Mo at any point




Deflection $y_{AC} = \dfrac{-M_0 x}{6LEI}(L^2 - 3b^2 - x^2)$

$y_{CB} = \dfrac{M_0(L-x)}{6LEI}(L^2 - 3a^2 - (L-x)^2)$

Slope $\theta_{AC} = \dfrac{-M_0}{6LEI}(L^2 - 3b^2 - 3x^2)$

$\theta_{CB} = \dfrac{M_0}{6LEI}(-L^2 + 3a^2 + 3(L-x)^2)$

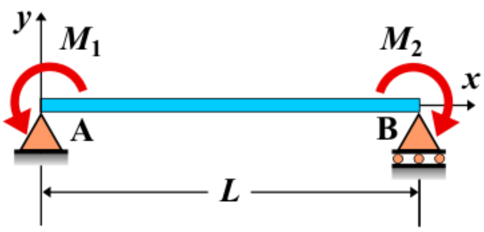
$\theta_A = \dfrac{-M_0}{6LEI}(L^2 - 3b^2) \qquad \theta_B = \dfrac{M_0}{6LEI}(-L^2 + 3a^2)$

Moment $M_{AC} = \dfrac{M_0 x}{L} \qquad M_{CB} = \dfrac{-M_0}{L}(L-x)$

Shear $V_{AC} = \dfrac{M_0}{L} \qquad V_{CB} = \dfrac{M_0}{L}$

Reactions $R_A = \dfrac{M_0}{L} \qquad R_B = \dfrac{-M_0}{L}$

## Simple beam - Couple moments $M_1$ and $M_2$ at each end (I)




Deflection $y_{AB} = \dfrac{-x(L-x)}{6LEI}\left[(M_1 - M_2)x - (2M_1 + M_2)L\right]$

Slope:

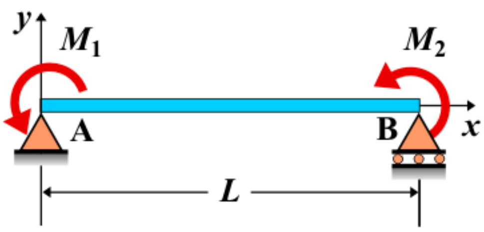
$\theta_{AB} = \dfrac{1}{6LEI}\left[(M_1 - M_2)(3x^2 - 2Lx) - (2M_1 + M_2)(2Lx - L^2)\right]$

Moment $M_{AB} = \dfrac{1}{L}\left[(M_1 - M_2)x - LM_1\right]$

Shear $V_{AB} = \dfrac{M_1 - M_2}{L}$

Reactions $R_A = \dfrac{M_1 - M_2}{L} \qquad R_B = \dfrac{M_2 - M_1}{L}$

## Simple beam - Couple moments $M_1$ and $M_2$ at each end (II)




Deflection $y_{AB} = \dfrac{-x(L-x)}{6LEI}\left[(M_1 + M_2)x - (2M_1 - M_2)L\right]$

Slope:

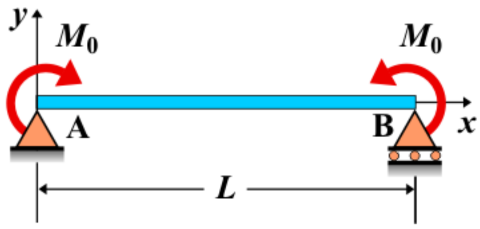
$\theta_{AB} = \dfrac{1}{6LEI}\left[(M_1 + M_2)(3x^2 - 2Lx) - (2M_1 - M_2)(2Lx - L^2)\right]$

Moment $M_{AB} = \dfrac{1}{L}\left[(M_1 + M_2)x - LM_1\right]$

Shear $V_{AB} = \dfrac{M_1 + M_2}{L}$

Reactions $R_A = \dfrac{M_1 + M_2}{L} \qquad R_B = \dfrac{-M_1 - M_2}{L}$

### Simple beam - Two equal couple moments Mo at each end




Deflection $y_{AB} = \frac{-M_0 x}{2EI}(L-x)$

$y_{MAX} = \frac{-M_0 L^2}{8EI}$ at $x = \frac{L}{2}$

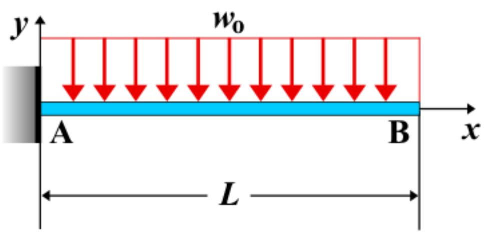
Slope $\theta_{AB} = \frac{-M_0}{2EI}(L-2x)$ $\quad \theta_A = -\theta_B = \frac{-M_0 L}{2EI}$

Moment $M_{AB} = M_0$

Shear $V_{AB} = 0$

Reactions $R_A = R_B = 0$

### Cantilever beam - Uniformly distributed load




Deflection $y_{AB} = \frac{-w_0}{24EI}(x^4 - 4Lx^3 + 6L^2x^2)$

$y_{MAX} = y_B = \frac{-w_0 L^4}{8EI}$ at $x = L$

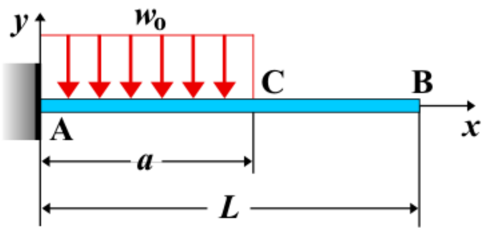
Slope $\theta_{AB} = \frac{-w_0}{6EI}(x^3 - 3Lx^2 + 3L^2x)$ $\quad \theta_B = \frac{-w_0 L^3}{6EI}$

Moment $M_{AB} = \frac{-w_0}{2}(L-x)^2$ $\quad M_{MAX} = M_A = \frac{-w_0 L^2}{2}$

Shear $V_{AB} = w_0(L-x)$

Reactions $R_A = w_0 L$

### Cantilever beam - Uniform load partially distributed at fixed end




Deflection $y_{AC} = \frac{-w_0}{24EI}(6a^2x^2 - 4ax^3 + x^4)$

$y_{CB} = \frac{-w_0 a^3}{24EI}(4x - a)$

$y_{MAX} = y_B = \frac{-w_0 a^3}{24EI}(4L - a)$

Slope $\theta_{AC} = \frac{-w_0}{6EI}(3a^2x - 3ax^2 + x^3)$

$\theta_{CB} = \theta_C = \theta_B = \frac{-w_0 a^3}{6EI}$

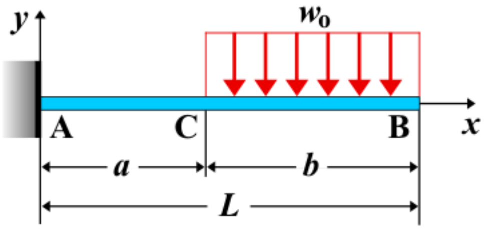
Moment $M_{AC} = \frac{-w_0}{2}(a-x)^2$ $\quad M_{CB} = M_C = M_B = 0$

$M_{MAX} = M_A = \frac{-w_0 a^2}{2}$

Shear $V_{AC} = w_0(a-x)$ $\quad V_{CB} = V_C = V_B = 0$

Reactions $R_A = w_0 a$

## Cantilever beam - Uniform load partially distributed at free end




Deflection $y_{AC} = \frac{-w_0 b x^2}{12EI}(3L + 3a - 2x)$

$y_{CB} = \frac{-w_0}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4)$

Slope $\theta_{AC} = \frac{-w_0 b x}{2EI}(L + a - x)$

$\theta_{CB} = \frac{-w_0}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3)$

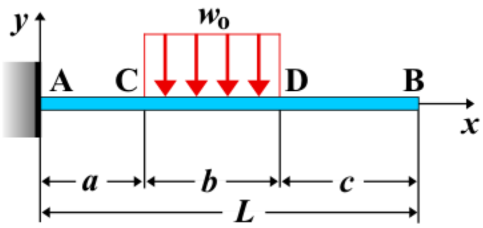
$\theta_B = \frac{-w_0}{6EI}(L^3 - a^3)$

Moment $M_{AC} = \frac{-w_0 b}{2}(L + a - 2x)$ $M_{CB} = \frac{-w_0}{2}(L - x)^2$

Shear $V_{AC} = V_A = V_C = w_0 b$ $V_{CB} = w_0(L - x)$

Reactions $R_A = w_0 b$

## Cantilever beam - Uniform load partially distributed




Deflection $y_{AC} = \frac{-w_0 b x^2}{12EI}(6a + 3b - 2x)$

$y_{CD} = \frac{-w_0}{24EI}(x^4 - 4(a+b)x^3 + 6(a+b)^2x^2 - 4a^3x + a^4)$

$y_{DB} = \frac{-w_0}{24EI}(4x[(a+b)^3 - a^3] - (a+b)^4 + a^4)$

Slope $\theta_{AC} = \frac{-w_0 b x}{2EI}(2a + b - x)$

$\theta_{CD} = \frac{-w_0}{6EI}(x^3 - 3(a+b)x^2 + 3(a+b)^2x - a^3)$

$\theta_{DB} = \frac{-w_0}{6EI}((a+b)^3 - a^3)$

Moment $M_{AC} = \frac{-w_0 b}{2}(2a + b - 2x)$

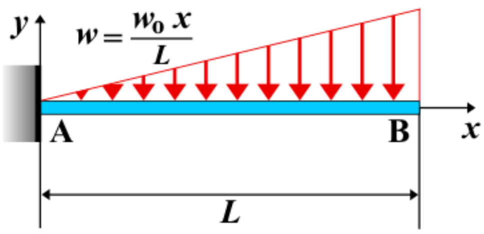
$M_{CD} = \frac{-w_0}{2}(a + b - x)^2$ $M_{DB} = M_D = M_B = 0$

Shear $V_{AC} = V_A = V_C = w_0 b$ $V_{CD} = w_0(a + b - x)$

$V_{DB} = V_D = V_B = 0$

Reactions $R_A = w_0 b$

### Cantilever beam - Load increasing uniformly to free end




Deflection $$y_{AB} = \frac{-w_0 x^2}{120LEI}(20L^3 - 10L^2 x + x^3)$$

$$y_{MAX} = \frac{-11 w_0 L^4}{120EI} \quad \text{at} \quad x = L$$

Slope $$\theta_{AB} = \frac{-w_0 x}{24LEI}(8L^3 - 6L^2 x + x^3)$$

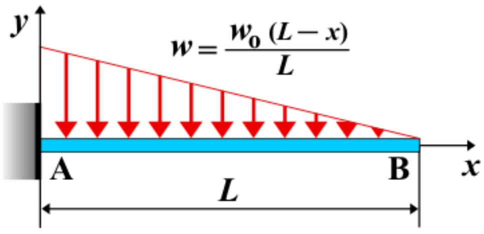
$$\theta_B = \frac{-w_0 L^3}{8EI}$$

Moment $$M_{AB} = \frac{-w_0}{6L}(2L^3 - 3L^2 x + x^3)$$

Shear $$V_{AB} = \frac{w_0}{2L}(L^2 - x^2)$$

Reactions $$R_A = \frac{w_0 L}{2}$$

### Cantilever beam - Load increasing uniformly to fixed end




Deflection $$y_{AB} = \frac{-w_0 x^2}{120LEI}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$$

$$y_{MAX} = \frac{w_0 L^4}{30EI} \quad \text{at} \quad x=L$$

Slope $$\theta_{AB} = \frac{-w_0 x}{24LEI}(4L^3 - 6L^2 x + 4Lx^2 - x^3)$$

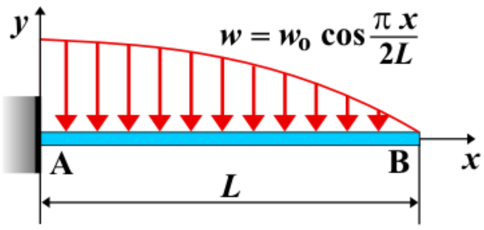
$$\theta_B = \frac{-w_0 L^3}{24EI}$$

Moment $$M_{AB} = \frac{-w_0}{6L}(L - x)^3$$

Shear $$V_{AB} = \frac{w_0}{2L}(L - x)^2$$

Reactions $$R_A = \frac{w_0 L}{2}$$

### Cantilever beam - Cosinusoidal distributed load




Deflection
$$y_{AB} = \frac{-w_0 L}{3\pi^4 EI}\left(48L^3 \cos\frac{\pi x}{2L} - 48L^3 + 3\pi^3 L x^2 - \pi^3 x^3\right)$$
$$y_{MAX} = \frac{-2w_0 L^4}{3\pi^4 EI}(\pi^3 - 24) \quad \text{at } x = L$$

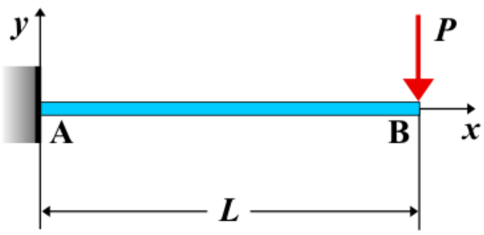
Slope
$$\theta_{AB} = \frac{-w_0 L}{\pi^3 EI}\left(2\pi^2 L x - \pi^2 x^2 - 8L^2 \sin\frac{\pi x}{2L}\right)$$
$$\theta_B = \frac{-w_0 L^3}{\pi^3 EI}(\pi^2 - 8)$$

Moment
$$M_{AB} = \frac{-2w_0 L}{\pi^2}\left(\pi L - \pi x - 2L\cos\frac{\pi x}{2L}\right)$$

Shear
$$V_{AB} = \frac{2w_0 L}{\pi}\left(1 - \sin\frac{\pi x}{2L}\right)$$

Reactions
$$R_A = \frac{2w_0 L}{\pi}$$

### Cantilever beam - Concentrated load P at free end




Deflection
$$y_{AB} = \frac{-P}{6EI}(3Lx^2 - x^3)$$
$$y_{MAX} = y_B = \frac{-PL^3}{3EI}$$

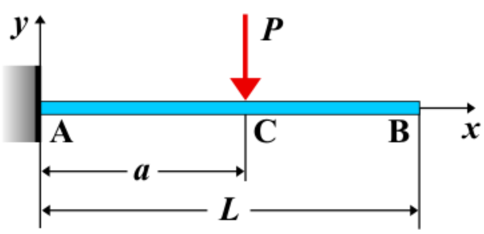
Slope
$$\theta_{AB} = \frac{-P}{2EI}(2Lx - x^2)$$
$$\theta_{MAX} = \theta_B = \frac{-PL^2}{2EI}$$

Moment $M_{AB} = -P(L - x)$ $\quad M_{MAX} = M_A = -PL$

Shear $V_{AB} = V_A = V_B = P$

Reactions $R_A = P$

### Cantilever beam - Concentrated load P at any point




Deflection
$$y_{AC} = \frac{-P}{6EI}(3ax^2 - x^3) \qquad y_{CB} = \frac{-Pa^2}{6EI}(3x - a)$$
$$y_{MAX} = y_B = \frac{-Pa^2}{6EI}(3L - a)$$

Slope
$$\theta_{AC} = \frac{-P}{2EI}(2ax - x^2) \qquad \theta_{CB} = \theta_C = \theta_B = \frac{-Pa^2}{2EI}$$

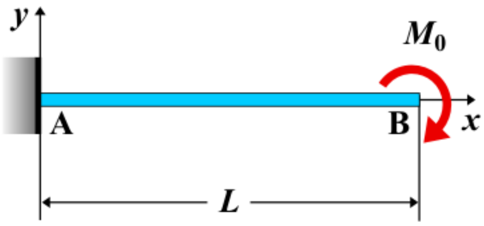
Moment $M_{AC} = -P(a - x)$ $\quad M_{CB} = M_C = M_B = 0$

$M_{MAX} = M_A = -Pa$

Shear $V_{AC} = V_A = V_C = P$ $\quad V_{CB} = V_C = V_B = 0$

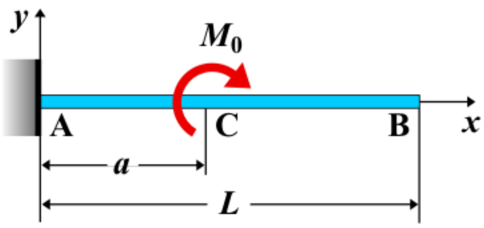
Reactions $R_A = P$

**Cantilever beam - Couple moment Mo at free end**



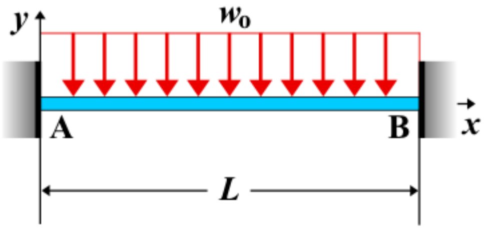

| | |
|---|---|
| Deflection | $y_{AB} = \dfrac{-M_0 x^2}{2EI}$ |
| | $y_{MAX} = \dfrac{-M_0 L^2}{2EI}$ at $x = L$ |
| Slope | $\theta_{AB} = \dfrac{-M_0 x}{EI}$ |
| Moment | $M_{AB} = M_A = M_B = -M_0$ |
| Shear | $V_{AB} = V_A = V_B = 0$ |
| Reactions | $R_A = 0$ |

**Cantilever beam - Couple moment Mo at any point**



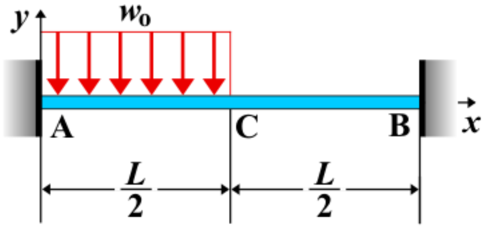

| | |
|---|---|
| Deflection | $y_{AC} = \dfrac{-M_0 x^2}{2EI}$ $\quad y_{CB} = \dfrac{-M_0 a}{2EI}(2x - a)$ |
| | $y_{MAX} = \dfrac{-M_0 a}{2EI}(2L - a)$ at $x = L$ |
| Slope | $\theta_{AC} = \dfrac{-M_0 x}{EI}$ $\quad \theta_{CB} = \theta_C = \theta_B = \dfrac{-M_0 a}{EI}$ |
| Moment | $M_{AC} = M_A = -M_0$ $\quad M_{CB} = M_B = 0$ |
| Shear | $V_{AC} = V_A = V_C = 0$ $\quad V_{CB} = V_C = V_B = 0$ |
| Reactions | $R_A = 0$ |

**Fixed-fixed beam - Uniformly distributed load**




| | |
|---|---|
| Deflection | $y_{AB} = \dfrac{-w_0 x^2}{24EI}(L - x)^2$ |
| Slope | $\theta_{AB} = \dfrac{-w_0 x}{12EI}(L^2 - 3Lx + 2x^2)$ |
| Moment | $M_{AB} = \dfrac{-w_0}{12}(L^2 - 6Lx + 6x^2)$ |
| Shear | $V_{AB} = \dfrac{w_0}{2}(L - 2x)$ |
| Reactions | $R_A = R_B = \dfrac{w_0 L}{2}$ |

**Fixed-fixed beam - Uniform load partially distributed at left end (I)**




Deflection $y_{AC} = \dfrac{-x^2}{24EI}(w_0x^2 - 4R_Ax - 12M_A)$

$$y_{CB} = \frac{3(M_B + LR_B)x^2 - R_Bx^3}{6EI} + \frac{L^2(3M_B + LR_B) - 3(2M_B + LR_B)Lx}{6EI}$$

Slope $\theta_{AC} = \dfrac{-x}{6EI}(w_0x^2 - 3R_Ax - 6M_A)$

$$\theta_{CB} = \frac{-1}{2EI}\left[R_Bx^2 - 2(M_B + LR_B)x + L(2M_B + LR_B)\right]$$

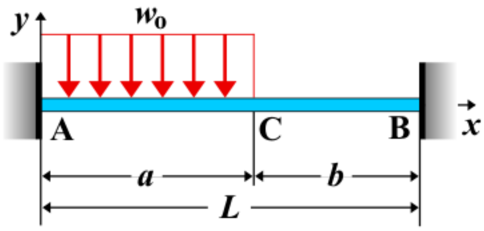
Moment $M_{AC} = R_Ax + M_A - \dfrac{w_0x^2}{2}$ $\quad M_{CB} = R_B(L-x) + M_B$

Shear $V_{AC} = R_A - w_0x$ $\quad V_{CB} = -R_B$

Reactions $R_A = \dfrac{3w_0L}{8} - \dfrac{M_A - M_B}{L}$ $\quad R_B = \dfrac{w_0L}{8} + \dfrac{M_A - M_B}{L}$

Where $M_A = \dfrac{-11w_0L^2}{192}$ $\quad M_B = \dfrac{-5w_0L^2}{192}$

**Fixed-fixed beam - Uniform load partially distributed at left end (II)**




Deflection $y_{AC} = \dfrac{-x^2}{24EI}(w_0x^2 - 4R_Ax - 12M_A)$

$$y_{CB} = \frac{3(M_B + LR_B)x^2 - R_Bx^3}{6EI} + \frac{L^2(3M_B + LR_B) - 3(2M_B + LR_B)Lx}{6EI}$$

Slope $\theta_{AC} = \dfrac{-x}{6EI}(w_0x^2 - 3R_Ax - 6M_A)$

$$\theta_{CB} = \frac{-1}{2EI}\left[R_Bx^2 - 2(M_B + LR_B)x + L(2M_B + LR_B)\right]$$

Moment $M_{AC} = R_Ax + M_A - \dfrac{w_0x^2}{2}$ $\quad M_{CB} = R_B(L-x) + M_B$

Shear $V_{AC} = R_A - w_0x$ $\quad V_{CB} = -R_B$

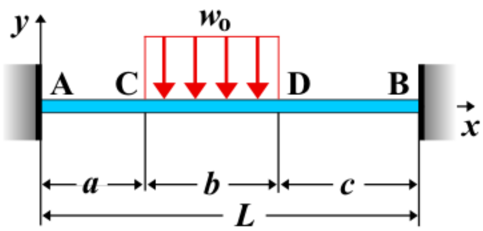
Reactions $R_A = \dfrac{w_0(L+b)a}{2L} - \dfrac{M_A - M_B}{L}$

$$R_B = \frac{w_0a^2}{2L} + \frac{M_A - M_B}{L}$$

Where $M_A = \dfrac{-w_0a^2}{12L^2}(6L^2 - 8La + 3a^2)$

$$M_B = \frac{-w_0a^3}{12L^2}(4L - 3a)$$

**Fixed-fixed beam - Uniform load partially distributed**




Deflection $y_{AC} = \frac{x^2}{6EI}(3M_A + R_A x)$

$$y_{CD} = \frac{-1}{24EI}\left[w_0 (x-a)^4 - 4R_A x^3 - 12M_A x^2\right]$$

$$y_{DB} = \frac{3(M_B + LR_B)x^2 - R_B x^3}{6EI} + \frac{L^2(3M_B + LR_B) - 3(2M_B + LR_B)Lx}{6EI}$$

Slope $\theta_{AC} = \frac{x}{2EI}(2M_A + R_A x)$

$$\theta_{CD} = \frac{-1}{6EI}\left[w_0 (x-a)^3 - 3R_A x^2 - 6M_A x\right]$$

$$\theta_{DB} = \frac{-1}{2EI}\left[R_B x^2 - 2(M_B + LR_B)x + L(2M_B + LR_B)\right]$$

Moment $M_{AC} = M_A + R_A x \quad M_{CD} = R_A x + M_A - \frac{w_0 (x-a)^2}{2}$

$$M_{DB} = M_B + R_B (L-x)$$

Shear $V_{AC} = R_A \quad V_{CD} = R_A - w_0 (x-a) \quad V_{DB} = -R_B$

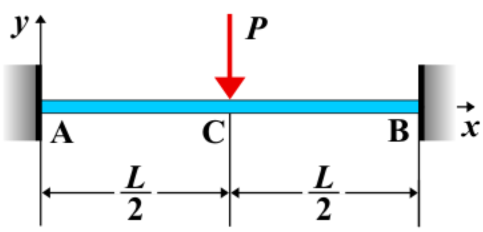
Reactions $R_A = \frac{w_0 (2c+b)b - 2M_A + 2M_B}{2L}$

$$R_B = \frac{w_0 (2a+b)b + 2M_A - 2M_B}{2L}$$

Where $M_A = \frac{-w_0 b}{24L^2}\left[b^2(2L - 6c - 3b) + (6a+3b)(2c+b)^2\right]$

$$M_B = \frac{-w_0 b}{24L^2}\left[b^2(2L - 6a - 3b) + (6c+3b)(2a+b)^2\right]$$

**Fixed-fixed beam - Concentrated load at center**




Deflection $y_{AC} = \frac{-Px^2}{48EI}(3L - 4x)$

$$y_{CB} = \frac{-P(L-x)^2}{48EI}(4x - L)$$

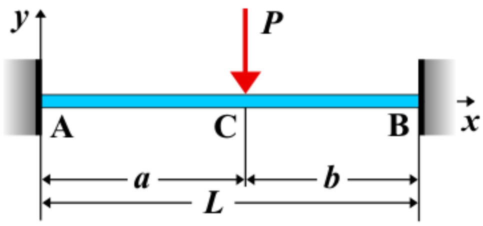
Slope $\theta_{AC} = \frac{-Px}{8EI}(L - 2x) \quad \theta_{CB} = \frac{-P}{8EI}(L^2 - 3Lx + 2x^2)$

Moment $M_{AC} = \frac{-P}{8}(L - 4x) \quad M_{CB} = \frac{P}{8}(3L - 4x)$

Shear $V_{AC} = \frac{P}{2} \quad V_{CB} = \frac{-P}{2}$

Reactions $R_A = R_B = \frac{P}{2}$

**Fixed-fixed beam - Concentrated load at any point**




Deflection $y_{AC} = \frac{-Pb^2x^2}{6EIL^3}(3aL - 3ax - bx)$

$y_{CB} = \frac{-Pa^2(L-x)^2}{6EIL^3}(3bx - aL + ax)$

Slope $\theta_{AC} = \frac{-Pb^2x}{2EIL^3}(2aL - 3ax - bx)$

$\theta_{CB} = \frac{Pa^2(L-x)}{2EIL^3}\left[x(3b+a) - L^2\right]$

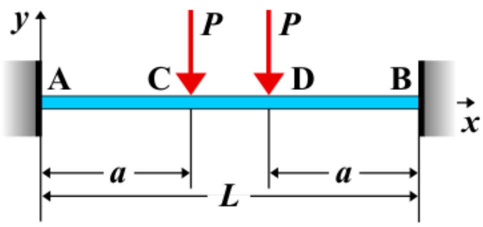
Moment $M_{AC} = \frac{-Pb^2x}{L^3}(aL - 3ax - bx)$

$M_{CB} = \frac{Pa^2}{L^3}(L^2 + bL - Lx - 2bx)$

Shear $V_{AC} = \frac{Pb^2}{L^3}(L+2a)$ $V_{CB} = \frac{-Pa^2}{L^3}(L+2b)$

Reactions $R_A = \frac{Pb^2}{L^3}(L+2a)$ $R_B = \frac{Pa^2}{L^3}(L+2b)$

**Fixed-fixed beam - Two equal concentrated loads symmetrically placed**




Deflection $y_{AC} = \frac{-Px^2}{6EIL}(3aL - 3a^2 - Lx)$

$y_{CD} = \frac{-Pa^2}{6EIL}(3Lx - 3x^2 - aL)$

$y_{DB} = \frac{-P(L-x)^2}{6EIL}(3aL - 3a^2 - L(L-x))$

Slope $\theta_{AC} = \frac{-Px}{2EIL}(2aL - 2a^2 - Lx)$ $\theta_{CD} = \frac{-Pa^2}{2EIL}(L-2x)$

$\theta_{DB} = \frac{P(L-x)}{2EIL}\left[2aL - 2a^2 - L(L-x)\right]$

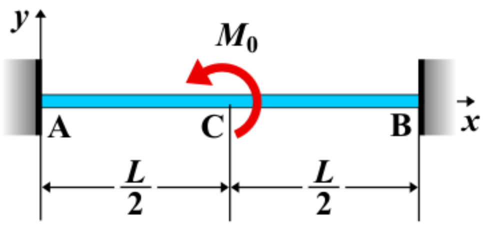
Moment $M_{AC} = \frac{P}{L}(Lx - aL + a^2)$

$M_{CD} = \frac{Pa^2}{L}$ $M_{DB} = \frac{P}{L}(L^2 - Lx - La + a^2)$

Shear $V_{AC} = P$ $V_{CD} = 0$ $V_{DB} = -P$

Reactions $R_A = R_B = P$

**Fixed-fixed beam - Couple moment Mo at center**




Deflection $y_{AC} = \frac{M_0 x^2}{8LEI}(2x - L)$

$y_{CB} = \frac{-M_0}{8LEI}(5Lx^2 - 2x^3 - 4L^2x + L^3)$

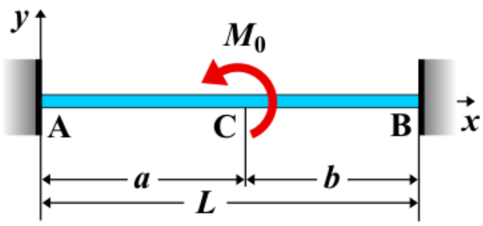
Slope $\theta_{AC} = \frac{M_0 x}{4LEI}(3x - L)$ $\theta_{CB} = \frac{-M_0}{8LEI}(10Lx - 6x^2 - 4L^2)$

Moment $M_{AC} = \frac{M_0}{4L}(6x - L)$ $M_{CB} = \frac{-M_0}{4L}(5L - 6x)$

Shear $V_{AB} = \frac{3M_0}{2L}$

Reactions $R_A = \frac{3M_0}{2L}$ $R_B = \frac{-3M_0}{2L}$

**Fixed-fixed beam - Couple moment Mo at any point**




Deflection:

$y_{AC} = \frac{-M_0 bx^2}{2L^3 EI}(2aL - 2ax - bL)$ $y_{CB} = \frac{M_0 a(L-x)^2}{2L^3 EI}(2bx - aL)$

Slope:

$\theta_{AC} = \frac{-M_0 bx}{L^3 EI}(2aL - 3ax - bL)$ $\theta_{CB} = \frac{M_0 a(L-x)}{L^3 EI}(L^2 - 3bx)$

Moment:

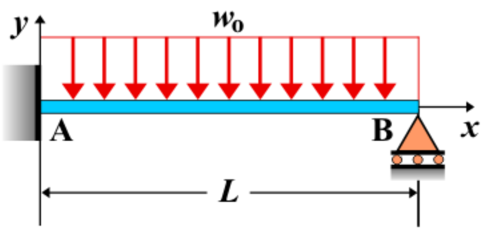
$M_{AC} = \frac{-M_0 b}{L^3}(2aL - 6ax - bL)$ $M_{CB} = \frac{M_0 a}{L^3}(6bx - 4bL - aL)$

Shear $V_{AB} = \frac{6M_0 ab}{L^3}$

Reactions $R_A = \frac{6M_0 ab}{L^3}$ $R_B = \frac{-6M_0 ab}{L^3}$

Where $M_A = \frac{-M_0 b}{L^2}(2a - b)$ $M_B = \frac{M_0 a}{L^2}(2b - a)$

**Fixed-pinned beam - Uniformly distributed load**




Deflection $y_{AB} = \frac{-w_0 x^2}{48EI}(3L^2 - 5Lx + 2x^2)$

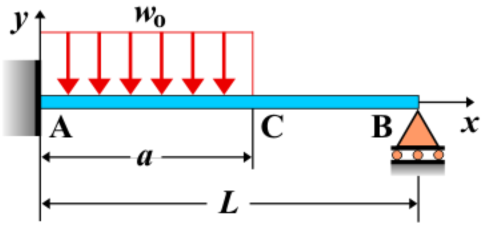
Slope $\theta_{AB} = \frac{-w_0 x}{48EI}(6L^2 - 15Lx + 8x^2)$

Moment $M_{AB} = \frac{-w_0}{8}(L^2 - 5Lx + 4x^2)$

Shear $V_{AB} = \frac{w_0}{8}(5L - 8x)$

Reactions $R_A = \frac{5w_0 L}{8}$ $R_B = \frac{3w_0 L}{8}$

**Fixed-pinned beam - Uniform load partially distributed at fixed end**




Deflection:

$$y_{AC} = \frac{8R_B L(L-x)^3 - 2w_0 L(a-x)^4 - w_0 a^3 (L-x)(L+3b)}{48EIL}$$

$$y_{CB} = \frac{8R_B L(L-x)^3 - w_0 a^3 (L-x)(L+3b)}{48EIL}$$

Slope:

$$\theta_{AC} = \frac{-24R_B L(L-x)^2 + 8w_0 L(a-x)^3 + w_0 a^3 (L+3b)}{48EIL}$$

$$\theta_{CB} = \frac{-24R_B L(L-x)^2 + w_0 a^3 (L+3b)}{48EIL}$$

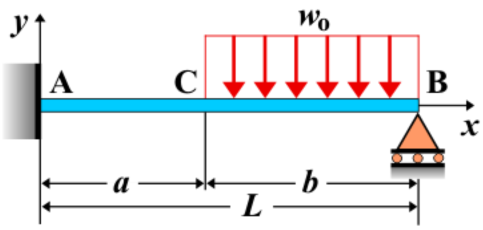
Moment $M_{AC} = \frac{2R_B(L-x) - w_0(a-x)^2}{2}$ $M_{CB} = R_B(L-x)$

Shear $V_{AC} = -R_B + w_0(a-x)$ $V_{CB} = -R_B$

Reactions $R_A = \frac{w_0(L+b)a - 2M_A}{2L}$ $R_B = \frac{w_0 a^2 + 2M_A}{2L}$

Where $M_A = \frac{-w_0(L+b)^2 a^2}{8L^2}$

**Fixed-pinned beam - Uniform load partially distributed at supported end**




Deflection $y_{AC} = \frac{x^2}{6EI}(R_A x + 3M_A)$

$$y_{CB} = \frac{4R_B L(L-x)^3 - w_0 L(L-x)^4}{24EIL} +$$

$$+ \frac{-w_0 b^2 (L-x)(bL + 3ab + 6a^2)}{48EIL}$$

Slope $\theta_{AC} = \frac{x}{2EI}(R_A x + 2M_A)$

$$\theta_{CB} = \frac{-3R_B L(L-x)^2 + w_0 L(L-x)^3}{6EIL} + \frac{w_0 b^2 (bL + 3ab + 6a^2)}{48EIL}$$

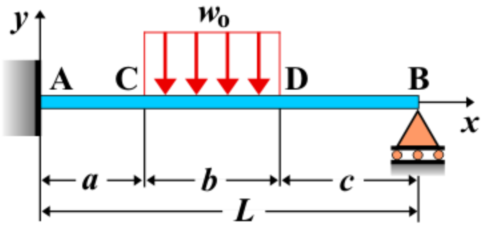
Moment $M_{AC} = R_A x + M_A$ $M_{CB} = \frac{2R_B(L-x) - w_0(L-x)^2}{2}$

Shear $V_{AC} = R_A$ $V_{CB} = -R_B + w_0(L-x)$

Reactions $R_A = \frac{w_0 b^2 - 2M_A}{2L}$ $R_B = \frac{w_0(2a+b)b + 2M_A}{2L}$

Where $M_A = \frac{-w_0 b^2}{16L^2}\left[(2L+b)(L+a) - b^2\right]$

## Fixed-pinned beam - Uniform load partially distributed




Deflection $$y_{AC} = \frac{x^2}{6EI}(R_A x + 3M_A)$$

$$y_{CD} = \frac{4R_B(L-x)^3 - w_0(L-x-c)^4}{24EI} + $$

$$+ \frac{-w_0 b(L-x)\left[2b^2L - 3b^2(2a+b) + 3(2c+b)(2a+b)^2\right]}{96EIL}$$

$$y_{DB} = \frac{R_B(L-x)^3}{6EI} + $$

$$+ \frac{-w_0 b(L-x)\left[2b^2L - 3b^2(2a+b) + 3(2c+b)(2a+b)^2\right]}{96EIL}$$

Slope $$\theta_{AC} = \frac{x}{2EI}(R_A x + 2M_A)$$

$$\theta_{CD} = \frac{-3R_B(L-x)^2 + w_0(L-x-c)^3}{6EI} + $$

$$+ \frac{w_0 b\left[2b^2L - 3b^2(2a+b) + 3(2c+b)(2a+b)^2\right]}{96EIL}$$

$$\theta_{DB} = \frac{-R_B(L-x)^2}{2EI} + $$

$$+ \frac{w_0 b\left[2b^2L - 3b^2(2a+b) + 3(2c+b)(2a+b)^2\right]}{96EIL}$$

Moment $$M_{AC} = R_A x + M_A$$

$$M_{CD} = \frac{2R_B(L-x) - w_0(L-x-c)^2}{2} \qquad M_{DB} = R_B(L-x)$$

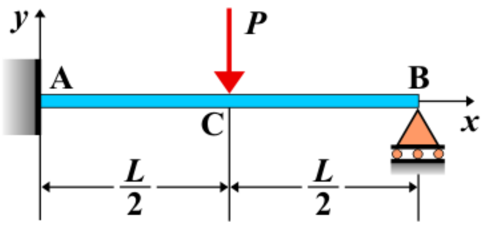
Shear $$V_{AC} = R_A \qquad V_{CD} = w_0(L-x-c) - R_B \qquad V_{DB} = -R_B$$

Reactions $$R_A = \frac{w_0 b(2c+b) - 2M_A}{2L}$$

$$R_B = \frac{w_0(2a+b)b + 2M_A}{2L}$$

Where $$M_A = \frac{-w_0(2c+b)(2a+b)b\left[(2L+2c+b)(2a+b) - b^2\right]}{16L^2}$$

#### Fixed-pinned beam - Concentrated load at center




Deflection:

$$y_{AC} = \frac{-Px^2}{96EI}(9L - 11x) \qquad y_{CB} = \frac{-P(L-x)}{96EI}(3L^2 - 5(L-x)^2)$$

Slope:

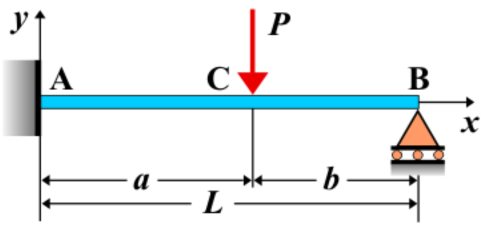
$$\theta_{AC} = \frac{-Px}{32EI}(6L - 11x) \qquad \theta_{CB} = \frac{-P}{32EI}(4L^2 - 10Lx + 5x^2)$$

Moment $\quad M_{AC} = \frac{-P}{16}(3L - 11x) \qquad M_{CB} = \frac{5P}{16}(L - x)$

Shear $\quad V_{AC} = \frac{11P}{16} \qquad V_{CB} = \frac{-5P}{16}$

Reactions $\quad R_A = \frac{11P}{16} \qquad R_B = \frac{5P}{16}$

#### Fixed-pinned beam - Concentrated load at any point




Deflection $\quad y_{AC} = \frac{-Pbx^2}{12EIL^3}(3L^3 - 3b^2L - 3L^2x + b^2x)$

$$y_{CB} = \frac{-Pa^2(L-x)}{12EIL^3}(3bL^2 - (2L+b)(L-x)^2)$$

Slope $\quad \theta_{AC} = \frac{-Pbx}{4EIL^3}(2L^3 - 2b^2L - 3L^2x + b^2x)$

$$\theta_{CB} = \frac{-Pa^2}{4EIL^3}(2L^3 - 4L^2x - 2bLx + 2Lx^2 + bx^2)$$

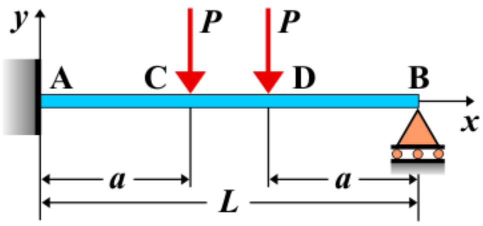
Moment:

$$M_{AC} = \frac{-Pb}{2L^3}(L^3 - b^2L - 3L^2x + b^2x) \quad M_{CB} = \frac{Pa^2}{2L^3}(L-x)(2L+b)$$

Shear $\quad V_{AC} = \frac{Pb}{2L^3}(3L^2 - b^2) \qquad V_{CB} = \frac{-Pa^2}{2L^3}(2L+b)$

Reactions $\quad R_A = \frac{Pb}{2L^3}(3L^2 - b^2) \qquad R_B = \frac{Pa^2}{2L^3}(2L+b)$

**Fixed-pinned beam - Two equal concentrated loads symmetrically placed**




Deflection:

$$y_{AC} = \frac{Px^2}{12EIL^2}\left[(3a^2 - 3aL - 2L^2)(L-x) + 2L(3a^2 - 3aL + L^2)\right]$$

$$y_{CD} = \frac{-Pa(3(L-a)(L-x)^3 - 6L^2(L-x)^2)}{12EIL^2} + \\ + \frac{-Pa\left[3L^2(L+a)(L-x) - 2L^2a^2\right]}{12EIL^2}$$

$$y_{DB} = \frac{-P(L-x)}{12EIL^2}\left[(3aL - 3a^2 - 2L^2)(L-x)^2 + 3aL^2(L-a)\right]$$

Slope:

$$\theta_{AC} = \frac{Px}{12EIL^2}\left[(3a^2 - 3aL - 2L^2)(2L-3x) + 4L(3a^2 - 3aL + L^2)\right]$$

$$\theta_{CD} = \frac{-Pa}{4EIL^2}\left[-3(L-a)(L-x)^2 + 4L^2(L-x) - L^2(L+a)\right]$$

$$\theta_{DB} = \frac{P}{4EIL^2}\left[(3aL - 3a^2 - 2L^2)(L-x)^2 + aL^2(L-a)\right]$$

Moment

$$M_{AC} = \frac{P}{2L^2}\left[3a^2L - 3aL^2 + x(2L^2 + 3aL - 3a^2)\right]$$

$$M_{CD} = \frac{-Pa}{2L^2}\left[3(L-a)(L-x) - 2L^2\right]$$

$$M_{DB} = \frac{-P(L-x)}{2L^2}(3aL - 3a^2 - 2L^2)$$

Shear

$$V_{AC} = \frac{P}{2L^2}(2L^2 + 3aL - 3a^2) \qquad V_{CD} = \frac{3Pa(L-a)}{2L^2}$$

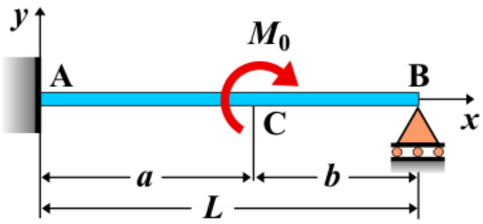
$$V_{DB} = \frac{P}{2L^2}(3aL - 3a^2 - 2L^2)$$

Reactions

$$R_A = \frac{P}{2L^2}(2L^2 + 3aL - 3a^2)$$

$$R_B = \frac{P}{2L^2}(3a^2 + 2L^2 - 3aL)$$

## Fixed-pinned beam - Couple moment Mo at any point




Deflection: $$y_{AC} = \frac{-M_0 x^2}{4EIL^3}\left[2b^2L - (L-x)(L^2-b^2)\right]$$

$$y_{CB} = \frac{-M_0 a(L-x)}{4EIL^3}\left[-4L^3 - ((L-x)^2 - 3L^2)(L+b)\right]$$

Slope $$\theta_{AC} = \frac{-M_0 x}{4EIL^3}\left[4b^2L - (2L-3x)(L^2-b^2)\right]$$

$$\theta_{CB} = \frac{-M_0 a}{4EIL^3}\left[4L^3 - 3(L+b)(x^2-2Lx)\right]$$

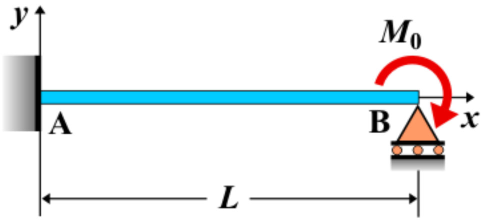
Moment $$M_{AC} = \frac{-M_0}{2L^3}\left[2b^2L - (L-3x)(L^2-b^2)\right]$$

$$M_{CB} = \frac{3M_0 a}{2L^3}(L+b)(L-x)$$

Shear $$V_{AB} = \frac{-3M_0 a}{2L^3}(L+b)$$

Reactions $$R_A = \frac{-3M_0 a}{2L^3}(L+b) \qquad R_B = \frac{3M_0 a}{2L^3}(L+b)$$

## Fixed-pinned beam - Couple moment Mo at supported end




Deflection $$y_{AB} = \frac{M_0 x^2 (L-x)}{4EIL}$$

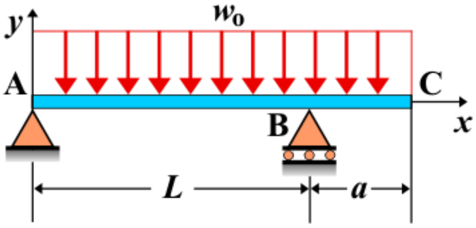
Slope $$\theta_{AB} = \frac{M_0 x(2L-3x)}{4EIL}$$

Moment $$M_{AB} = \frac{M_0 (L-3x)}{2L}$$

Shear $$V_{AB} = \frac{-3M_0}{2L}$$

Reactions $$R_A = \frac{-3M_0}{2L} \qquad R_B = \frac{3M_0}{2L}$$

### Overhanging beam - Uniformly distributed load




Deflection $y_{AB} = \dfrac{-w_0 x}{24LEI}(L^4 - 2L^2x^2 + Lx^3 - 2a^2L^2 + 2a^2x^2)$

$y_{BC} = \dfrac{-w_0 x_1}{24EI}(4a^2L - L^3 + 6a^2x_1 - 4ax_1^{\,2} + x_1^{\,3})$

Slope $\theta_{AB} = \dfrac{-w_0}{24LEI}(L^4 - 6L^2x^2 + 4Lx^3 - 2a^2L^2 + 6a^2x^2)$

$\theta_{BC} = \dfrac{-w_0}{24EI}(4a^2L - L^3 + 12a^2x_1 - 12ax_1^{\,2} + 4x_1^{\,3})$

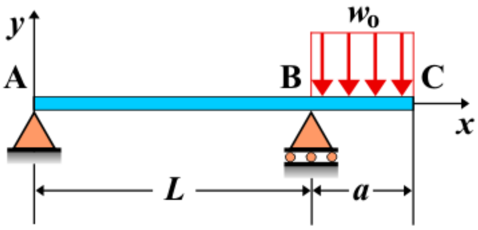
Moment $M_{AB} = \dfrac{w_0 x}{2L}(L^2 - Lx - a^2)$ $M_{BC} = \dfrac{-w_0}{2}(a - x_1)^2$

Shear $V_{AB} = \dfrac{w_0}{2L}(L^2 - 2Lx - a^2)$ $V_{BC} = w_0(a - x_1)$

Reactions $R_A = \dfrac{w_0}{2L}(L^2 - a^2)$ $R_B = \dfrac{w_0}{2L}(L + a)^2$

Where $x_1 = x - L$

### Overhanging beam - Uniformly distributed load on overhang




Deflection $y_{AB} = \dfrac{w_0 a^2 x}{12LEI}(L^2 - x^2)$

$y_{BC} = \dfrac{-w_0 x_1}{24EI}(4a^2L + 6a^2x_1 - 4ax_1^{\,2} + x_1^{\,3})$

Slope $\theta_{AB} = \dfrac{w_0 a^2}{12LEI}(L^2 - 3x^2)$

$\theta_{BC} = \dfrac{-w_0}{6EI}(a^2L + 3a^2x_1 - 3ax_1^{\,2} + x_1^{\,3})$

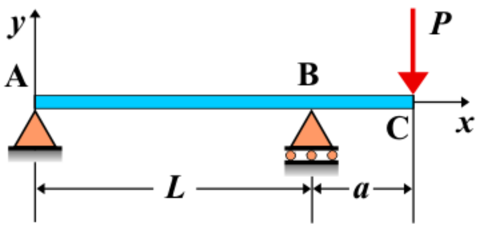
Moment $M_{AB} = \dfrac{-w_0 a^2 x}{2L}$ $M_{BC} = \dfrac{-w_0}{2}(a - x_1)^2$

Shear $V_{AB} = \dfrac{-w_0 a^2}{2L}$ $V_{BC} = w_0(a - x_1)$

Reactions $R_A = \dfrac{-w_0 a^2}{2L}$ $R_B = \dfrac{w_0(2L + a)a}{2L}$

Where $x_1 = x - L$

**Overhanging beam - Concentrated load at end of overhang**




Deflection $y_{AB} = \frac{Pax}{6LEI}(L^2 - x^2)$

$y_{BC} = \frac{-Px_1}{6EI}(2aL + 3ax_1 - x_1^2)$

Slope $\theta_{AB} = \frac{Pa}{6LEI}(L^2 - 3x^2)$ $\theta_{BC} = \frac{-P}{6EI}(2aL + 6ax_1 - 3x_1^2)$

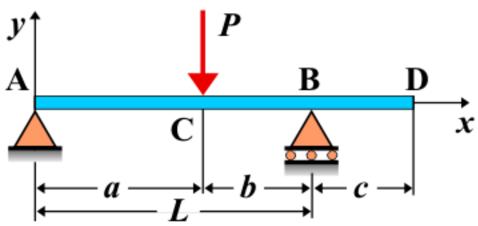
Moment $M_{AB} = \frac{-Pax}{L}$ $M_{BC} = -P(a - x_1)$

Shear $V_{AB} = \frac{-Pa}{L}$ $V_{BC} = P$

Reactions $R_A = \frac{-Pa}{L}$ $R_B = \frac{P(L+a)}{L}$

Where $x_1 = x - L$

**Overhanging beam - Concentrated load at any point between supports**




Deflection $y_{AC} = \frac{-Pbx}{6LEI}(L^2 - b^2 - x^2)$

$y_{CB} = \frac{-Pa(L-x)}{6LEI}(2Lx - a^2 - x^2)$

$y_{BD} = \frac{Pabx_1}{6LEI}(L + a)$

Slope $\theta_{AC} = \frac{-Pb}{6LEI}(L^2 - b^2 - 3x^2)$

$\theta_{CB} = \frac{-Pa}{6LEI}(2L^2 - 6Lx + a^2 + 3x^2)$ $\theta_{BD} = \frac{Pab(L+a)}{6LEI}$

Moment $M_{AC} = \frac{Pbx}{L}$ $M_{CB} = \frac{Pa}{L}(L - x)$ $M_{BD} = 0$

Shear $V_{AC} = \frac{Pb}{L}$ $V_{CB} = \frac{-Pa}{L}$ $V_{BD} = 0$

Reactions $R_A = \frac{Pb}{L}$ $R_B = \frac{Pa}{L}$

Where $x_1 = x - L$