Newton's binomial theorem and Pascal-Tartaglia's triangle

Newtons's binomial theorem

The expansion of the binomial power $(a+b)^n$ is:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}\, b^k$$

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \ldots + \binom{n}{n-2} a^2\, b^{n-2} + \binom{n}{n-1} a\, b^{n-1} + \binom{n}{n} b^n$$

Whenever n is positive integer number.

The binomial coeficients are:

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

Where $n!$ denotes factorial of n

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 4 \cdot 3 \cdot 2 \cdot 1$. Example: $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

Examples:

$$(a + b)^2 = a^2 + 2\,a\,b + b^2$$
$$(a - b)^2 = a^2 - 2\,a\,b + b^2$$

$$(a + b)^3 = a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3$$
$$(a - b)^3 = a^3 - 3\,a^2\,b + 3a\,b^2 - b^3$$

$$(a + b)^4 = a^4 + 4a^3\,b + 6a^2\,b^2 + 4a\,b^3 + b^4$$
$$(a - b)^4 = a^4 - 4a^3\,b + 6a^2\,b^2 - 4a\,b^3 + b^4$$

$$(a + b)^5 = a^5 + 5a^4\,b + 10a^3\,b^2 + 10a^2\,b^3 + 5a\,b^4 + b^5$$
$$(a - b)^5 = a^5 - 5a^4\,b + 10a^3\,b^2 - 10a^2\,b^3 + 5a\,b^4 - b^5$$

Pascal-Tartaglia's triangle

This triangle determines the coefficients which arise in binomial expansions:

n											
0						1					
1					1		1				
2				1		2		1			
3			1		3		3		1		
4		1		4		6		4		1	
5	1		5		10		10		5		1
6		1		6		15		20		15		6		1	
7			1		7		21		35		35		21		7		1	
8		1		8		28		56		70		56		28		8		1	
9	1		9		36		84		126		126		84		36		9		1
10	1		10		45		120		210		252		210		120		45		10		1

On the zeroth row, write only the number 1.
Then, to construct the elements of following rows, add the number directly above and to the left with the number directly above and to the right to find the new value.