

Classification and reduced equations of conic sections

Conic sections have the form of a second-degree polynomial:

$$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{01}x + 2a_{02}y + a_{00} = 0$$

We can calculate the following determinants:

$$|A| = \begin{vmatrix} a_{00} & a_{01} & a_{02} \\ a_{01} & a_{11} & a_{12} \\ a_{02} & a_{12} & a_{22} \end{vmatrix}, \quad A_{00} = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}, \quad A_{11} = \begin{vmatrix} a_{00} & a_{02} \\ a_{02} & a_{11} \end{vmatrix}, \quad A_{22} = \begin{vmatrix} a_{00} & a_{01} \\ a_{01} & a_{11} \end{vmatrix}$$

Relationship between the coefficients	Classification: regular conic section	Reduced equation	Coefficients of the reduced equation
$ A \neq 0$	$A_{00} > 0$	$b_{11}x^2 + b_{22}y^2 + b_{00} = 0$	$b_{00} = \frac{ A }{A_{00}}$ b_{11}, b_{22} are the roots of : $t^2 - (a_{11} + a_{22})t + A_{00} = 0$
	$(a_{11} + a_{22}) \cdot A < 0$ ELLIPSE (CIRCLE if $a_{11} = a_{22}, a_{12} = 0$)		
	$(a_{11} + a_{22}) \cdot A > 0$ IMAGINARY ELLIPSE		
$A_{00} < 0$	HYPERBOLA (Equilateral hyperbola if $a_{11} + a_{22} = 0$)		
$A_{00} = 0$	PARABOLA	$b_{22}y^2 + 2b_{01}x = 0$	$b_{22} = a_{11} + a_{22}$ $b_{01} = \pm \sqrt{\frac{- A }{a_{11} + a_{22}}}$

Relationship between the coefficients	Classification: degenerated conic section	Reduced equation	Coefficients of the reduced equation	
$ A = 0$	$A_{00} > 0$	TWO IMAGINARY INTERSECTING LINES	b_{11}, b_{22} are the roots of : $t^2 - (a_{11} + a_{22})t + A_{00} = 0$	
	$A_{00} < 0$	TWO REAL INTERSECTING LINES		
	$A_{00} = 0$	$A_{11} > 0$ or $A_{22} > 0$	TWO IMAGINARY PARALLEL LINES	$b_{22} = a_{11} + a_{22}$ $b_{00} = \frac{A_{11} + A_{22}}{a_{11} + a_{22}}$
		$A_{11} < 0$ or $A_{22} < 0$	TWO REAL PARALLEL LINES	
$A_{11} = A_{22} = 0$	A SINGLE DOUBLED LINE	$y^2 = 0$		