Classification and reduced equations of conic sections

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Conic sections have the form of a second-degree polynomial:

$$a_{11}x^2 + a_{22}y^2 + 2a_{12}xy + 2a_{01}x + 2a_{02}y + a_{00} = 0$$

We can calculate the following determinants:

$$|A| = \begin{vmatrix} a_{00} & a_{01} & a_{02} \\ a_{01} & a_{11} & a_{12} \\ a_{02} & a_{12} & a_{22} \end{vmatrix}, \quad A_{00} = \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}, \quad A_{11} = \begin{vmatrix} a_{00} & a_{02} \\ a_{02} & a_{11} \end{vmatrix}, \quad A_{22} = \begin{vmatrix} a_{00} & a_{01} \\ a_{01} & a_{11} \end{vmatrix}$$

Relationship between the coefficients	Classification: regular conic section	Reduced equation	Coefficients of the reduced equation
$\|A\| \neq 0$, $A_{00} > 0$, $(a_{11}+a_{22})\cdot\|A\| < 0$	ELLIPSE (CIRCLE if $a_{11} = a_{22}$, $a_{12} = 0$)	$b_{11}x^2 + b_{22}y^2 + b_{00} = 0$	$b_{00} = \dfrac{\|A\|}{A_{00}}$; b_{11}, b_{22} are the roots of: $t^2 - (a_{11}+a_{22})t + A_{00} = 0$
$\|A\| \neq 0$, $A_{00} > 0$, $(a_{11}+a_{22})\cdot\|A\| > 0$	IMAGINARY ELLIPSE	$b_{11}x^2 + b_{22}y^2 + b_{00} = 0$	$b_{00} = \dfrac{\|A\|}{A_{00}}$; b_{11}, b_{22} are the roots of: $t^2 - (a_{11}+a_{22})t + A_{00} = 0$
$\|A\| \neq 0$, $A_{00} < 0$	HYPERBOLA (Equilateral hypebola if $a_{11} + a_{22} = 0$)	$b_{11}x^2 + b_{22}y^2 + b_{00} = 0$	$b_{00} = \dfrac{\|A\|}{A_{00}}$; b_{11}, b_{22} are the roots of: $t^2 - (a_{11}+a_{22})t + A_{00} = 0$
$A_{00} = 0$	PARABOLA	$b_{22}y^2 + 2b_{01}x = 0$	$b_{22} = a_{11} + a_{22}$; $b_{01} = \pm\sqrt{\dfrac{-\|A\|}{a_{11}+a_{22}}}$

Relationship between the coefficients	Classification: degenerated conic section	Reduced equation	Coefficients of the reduced equation
$\lvert A\rvert = 0$, $A_{00} > 0$	TWO IMAGINARY INTERSECTING LINES	$b_{11}x^2 + b_{22}y^2 = 0$	b_{11}, b_{22} are the roots of : $t^2 - (a_{11} + a_{22})t + A_{00} = 0$
$\lvert A\rvert = 0$, $A_{00} < 0$	TWO REAL INTERSECTING LINES		
$\lvert A\rvert = 0$, $A_{00} = 0$, $A_{11} > 0$ or $A_{22} > 0$	TWO IMAGINARY PARALLEL LINES	$b_{22}y^2 + b_{00} = 0$	$b_{22} = a_{11} + a_{22}$, $b_{00} = \frac{A_{11} + A_{22}}{a_{11} + a_{22}}$
$\lvert A\rvert = 0$, $A_{00} = 0$, $A_{11} < 0$ or $A_{22} < 0$	TWO REAL PARALLEL LINES		
$\lvert A\rvert = 0$, $A_{00} = 0$, $A_{11} = A_{22} = 0$	A SINGLE DOUBLED LINE	$y^2 = 0$	