Del operator (nabla operator)

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{\mathrm{i}} + \frac{\partial}{\partial y}\vec{\mathrm{j}} + \frac{\partial}{\partial z}\vec{\mathrm{k}}$$ (three-dimensional cartesian coordinate system)

Gradient of a scalar function *f*

$$\mathrm{grad}\ f = \vec{\nabla} f = \frac{\partial f}{\partial x}\vec{\mathrm{i}} + \frac{\partial f}{\partial y}\vec{\mathrm{j}} + \frac{\partial f}{\partial z}\vec{\mathrm{k}}$$

Where:
$f = f(x, y, z)$ is a scalar function of x, y, z.
$\vec{\nabla} f$ = gradient of f, a vector function.

Divergence of a vector field $\vec{v}$

$$\mathrm{div}\,\vec{v} = \vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Where:
$\vec{v}$ is a vector function (vector field) of x, y, z.
$\vec{v} = v_x(x,y,z)\vec{\mathrm{i}} + v_y(x,y,z)\vec{\mathrm{j}} + v_z(x,y,z)\vec{\mathrm{k}}$
$\vec{\nabla}\cdot\vec{v}$ = divergence of $\vec{v}$, a scalar function.

Curl of a vector field $\vec{v}$

$$\mathrm{curl}\,\vec{v} = \vec{\nabla}\times\vec{v} = \begin{vmatrix} \vec{\mathrm{i}} & \vec{\mathrm{j}} & \vec{\mathrm{k}} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\vec{\mathrm{i}} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\vec{\mathrm{j}} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\vec{\mathrm{k}}$$

Where:
$\vec{v}$ is a vector function (vector field) of x, y, z.
$\vec{v} = v_x(x,y,z)\vec{\mathrm{i}} + v_y(x,y,z)\vec{\mathrm{j}} + v_z(x,y,z)\vec{\mathrm{k}}$
$\vec{\nabla}\times\vec{v}$ = curl of $\vec{v}$, a vector function.

Directional derivative of a scalar function *f* in the direction of the vector $\vec{v}$

$$\mathrm{D}_{\vec{v}}\ f = \vec{\nabla} f \cdot \frac{\vec{v}}{|\vec{v}|}$$

Where:
$f = f(x, y, z)$ is a scalar function of x, y, z.
$\vec{\nabla} f$ = gradient of f, a vector function.
$\vec{v}$ is a vector.

Tangent plane to a surface *f*

$$f_x(p_0)(x - x_0) + f_y(p_0)(y - y_0) + f_z(p_0)(z - z_0) = 0$$

$$\vec{\nabla} f(p_0)\cdot(\vec{r} - \overrightarrow{Op_0}) = 0$$

Where:
$f = f(x, y, z)$ is a scalar function of x, y, z.
$p_0(x_0, y_0, z_0)$ a point on the surface f.
$f_x(p_0)$= Partial derivative of f with respect to x evaluated at the point $p_0(x_0, y_0, z_0)$, etc.
$\vec{r} = x\vec{\mathrm{i}} + y\vec{\mathrm{j}} + z\vec{\mathrm{k}}$