

Separable equations	$F(x) G(y) dx + H(x) P(y) dy = 0$
Solution	$\int \frac{F(x)}{H(x)} dx + \int \frac{P(y)}{G(y)} dy = K$
Homogeneous nonlinear equations	$M(x, y) dx + N(x, y) dy = 0$
Test	$M(\lambda x, \lambda y) = \lambda^n M(x, y) \wedge N(\lambda x, \lambda y) = \lambda^n N(x, y)$
Procedure	Substitution $y = v x, \quad f(v) = -\frac{M(x, v)}{N(x, v)}$
Solution	$x = K e^{\int \frac{dv}{f(v)-v}}$ then we make the change of variable $v = y / x$
Exact equations	$M(x, y) dx + N(x, y) dy = 0$
Test for exactness	$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
Solution	$\int M dx + \int \left(N - \frac{\partial}{\partial y} \int M dx \right) dy = K \quad \text{or ...}$ $\int N dy + \int \left(M - \frac{\partial}{\partial x} \int N dy \right) dx = K$
Reducible to exact equations (Integrating factors)	$M(x, y) dx + N(x, y) dy = 0$
Test	if $p(x, y) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}$ depends only on $x \quad \mu(x) = e^{\int p(x) dx}$ if $q(x, y) = \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}$ depends only on $y \quad \mu(y) = e^{\int q(y) dy}$
Solution	Transform given equation into an exact differential multiplying it both sides by integrating factor $\mu(x)$ or $\mu(y)$.
Nonhomogeneous linear equations	$y' + p(x) y = q(x)$
Solution	$y e^{\int p dx} = \int q e^{\int p dx} dx + K$
Homogeneous linear equations	$y' + p(x) y = 0$
Solution	$y e^{\int p dx} = K$
Bernoulli equations	$y' + p(x) y = q(x) y^n \quad \text{where } n \neq 0 \wedge n \neq 1$
Solution	$y^{1-n} e^{\int (1-n)p dx} = \int (1-n)q e^{\int (1-n)p dx} dx + K$ If $n = 0$, view <i>Nonhomogeneous linear equations</i> . If $n = 1$, view <i>Homogeneous linear equations</i> .