

Taylor series of a function of one variable.

The Taylor series of a function $f(x)$ that is infinitely differentiable in a neighbourhood of number a , is the power series:

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \dots + \frac{f^n(a)(x-a)^n}{n!} + \dots$$

Where $f^n(a)$ denotes the n th derivative of the function at the point a .

$n!$ is the factorial of n .

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \dots \cdot 4 \cdot 3 \cdot 2 \cdot 1$.

Example: $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

If the series uses the derivatives at zero, the series is also called a Maclaurin series.

Taylor series of some common functions.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad -\infty < x < \infty$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \dots \quad -1 < x \leq 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 \dots \quad -1 < x < 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad -\infty < x < \infty$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \quad -\infty < x < \infty$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \quad -\infty < x < \infty$$