

Taylor series of a real function in one variable.

The Taylor series of a function $f(x)$ that is infinitely differentiable in a neighbourhood of number a , is the power series:

$$f(x) = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!} + \dots + \frac{f^n(a)(x-a)^n}{n!} + \dots$$

Where $f^n(a)$ denotes the n th derivative of the function at the point a .

$n!$ is the factorial of n .

$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \dots \cdot 4 \cdot 3 \cdot 2 \cdot 1$.

Example: $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

If the series uses the derivatives at zero, the series is also called a Maclaurin series.

Taylor series of some common functions.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad -\infty < x < \infty$$

$$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \frac{(x-1)^5}{5} - \dots \quad 0 < x \leq 2$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \quad -1 < x \leq 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \dots \quad -1 < x < 1$$

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + 5x^4 - 6x^5 + \dots \quad -1 < x < 1$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + \dots \quad -1 < x < 1$$

$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \dots \quad -1 < x < 1$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \dots \quad -1 < x \leq 1$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad -\infty < x < \infty$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots \quad -\infty < x < \infty$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \quad -\infty < x < \infty$$

$$\arcsin x = x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \quad -1 < x < 1$$

$$\arccos x = \frac{\pi}{2} - \arcsin x = \frac{\pi}{2} - \left(x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \frac{x^7}{7} + \dots \right) \quad -1 < x < 1$$

$$\arctan x = \begin{cases} -\frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots & x \leq -1 \\ x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots & -1 < x < 1 \\ \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} - \frac{1}{5x^5} + \dots & x \geq 1 \end{cases}$$