Problems of Analytic geometry of Space

1) Use the triple product to find out the volume of the parallelepiped determined by the vectors $\vec{a}(6,\ 8,\ 7)$, $\vec{b}(13,\ -7,\ 8)$ and $\vec{c}(4,\ 6,\ 2)$.

2) Consider the vectors $\vec{u}(28,\ 7,\ 16)$ and $\vec{v}(3,\ 16,\ -24)$. Find out:
a) Magnitude of each vector. **b)** Scalar product (dot product). **c)** Angle between these vectors.

3) Calculate the angle between the following pairs of vectors:
a) $\vec{u}(1,\ -11,\ 9)$ and $\vec{v}(13,\ 16,\ 13)$ **b)** $\vec{w}(11,\ 1,\ -19)$ and $\vec{a}(15,\ 13,\ -6)$

4) Find out the unit vector of each of the following vectors:
a) $\vec{u}(13,\ -14,\ -34)$ **b)** $\vec{v}(20,\ 22,\ 4)$

5) Consider the vectors $\vec{u}(8,\ 7,\ -8)$, $\vec{v}(7,\ -9,\ 1)$, $\vec{w}(-8,\ 1,\ 6)$ and $\vec{a}(-49,\ 6,\ 41)$.
Find out the value of $x,\ y,\ z$ for which verify the following linear combination: $\vec{a} = x\,\vec{u} + y\,\vec{v} + z\,\vec{w}$.

6) Consider the vectors $\vec{u}(8,\ 2,\ p)$ and $\vec{v}(k,\ 12,\ -15)$. Find out the values of parameters p and k for which the vectors are perpendicular (orthogonal) and the magnitude of $\vec{v}$ is 25 if $k > 0$.

7) Find out a unit vector perpendicular to $\vec{u}(4,\ -2,\ -4)$ and $\vec{v}(2,\ 12,\ 11)$.

8) Consider the vectors $\vec{a}(9,\ r,\ 24)$ and $\vec{b}(27,\ -5,\ 72)$. Find out the value(s) of parameter r for each of the following cases:
a) Magnitude of vector $\vec{a}$ is 41. **b)** Vectors are parallel. **c)** Vectors are orthogonal (perpendicular).

9) Find out the value of parameter k for which vectors $\vec{a}(13,\ 5,\ -12)$, $\vec{b}(3,\ 14,\ 13)$ and $\vec{c}(k,\ 12,\ 12)$ are linearly independent.

10) Find out the equation of the plane that passes through the point $A(7, 5, -2)$ and is parallel to the plane $\pi: 8x - 3y - 11z - 68 = 0$.

11) Find out the equation of the plane that contains the point $A(-6, -6, -5)$ and is parallel to vectors $\vec{v}(3, 3, 1)$ and $\vec{u}(4, 3, 3)$.

12) Find out the equation of the plane that passes through the points $A(4, 2, -7)$ and $B(4, 9, -15)$ and is parallel to the vector $\vec{u}(-8, 3, 0)$.

13) Find out the equation of the plane that passes through the point $B(-2, 0, -1)$ and contains the line $L: (x, y, z) = (-5, -1, -7) + t\,(3, 7, 2)$.

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14) Find out the equation of the plane that passes through the points $A(1, 5, -4)$ and $B(5, 10, 0)$ and is parallel to the line $L: \begin{cases} 8y + 5z - 32 = 0 \\ 8x - 5z - 16 = 0 \end{cases}$.

15) Find out the equation of the plane that contains the point $A(0, 4, 10)$ and is perpendicular to the vector $\vec{v}(11, -4, -4)$.

16) Find out the equation of the plane that contains the parallel lines $L_1: \begin{cases} x = 1 + 3t \\ y = 2 + t \\ z = 2 + t \end{cases}$ and $L_2: \dfrac{x-2}{3} = \dfrac{y-10}{1} = \dfrac{z-10}{1}$.

17) Find out the equation of the plane that contains the line $L_1: \begin{cases} -y - 3z - 7 = 0 \\ -x - z - 5 = 0 \end{cases}$ and is parallel to the line $L_2: \dfrac{x-3}{0} = \dfrac{y-1}{7} = \dfrac{z-6}{-6}$.

18) Find out the equation of the plane that contains the point $A(11, 4, 2)$ and is parallel to the lines $L_1: (x, y, z) = (1, -2, 2) + t\,(4, 3, 4)$ and $L_2: \dfrac{x-1}{3} = \dfrac{y+2}{2} = \dfrac{z+1}{3}$.

19) Find out the equation of the plane that contains the points $A(-2, 9, 3)$ and $B(-1, 9, 4)$ and is perpendicular to the plane $\pi: 3x - y + 2z + 21 = 0$.

20) Find out the symmetric form equation of the line that contains the point $A(-3, -5, 12)$ and is parallel to the line $L: \begin{cases} 9y - 9z - 9 = 0 \\ 9x + 4z + 5 = 0 \end{cases}$.

21) Find out the symmetric form equation of the line that contains the point $A(10, -7, 11)$ and is parallel to the vector $\vec{v}(-1, 2, 4)$.

22) Find out the symmetric form equation of the line that contains the point $A(8, -3, 0)$ and is perpendicular to the plane $\pi: 10x + 7y + z + 4 = 0$.

23) Find out the symmetric form equation of the line that passes through the point $A(8, 1, -1)$, is parallel to

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24) Find out the symmetric form equation of the line that is parallel to the plane $\pi: 5x + 4y + 2z + 11 = 0$, is perpendicular to the line $L: \frac{x+4}{-2} = \frac{y+5}{1} = \frac{z+9}{3}$ and cuts this line at the point $A(-8, -3, -3)$.

25) Find out the symmetric form equation of the line that contains the point $A(5, -9, -7)$ and cuts the lines $L_1: \frac{x-2}{-7} = \frac{y+12}{3} = \frac{z+13}{-4}$ and $L_2: \frac{x-12}{-3} = \frac{y+22}{2} = \frac{z+5}{7}$.

26) Find out the symmetric form equation of the line that is parallel to the plane $\pi: 4x + 4y - 4z - 2 = 0$, is perpendicular to the line $L: \frac{x+8}{8} = \frac{y+11}{4} = \frac{z-1}{-7}$ and cuts this line at the point $A(0, -7, -6)$.

27) Calculate the value of parameter m for which the points $A\,(7,\ 5,\ -4)$, $B\,(8,\ 0,\ 0)$, and $C\,(m,\ -5,\ 4)$ are aligned.

28) Find out the values of parameter r for which the distance between the point $A\,(6,\ r,\ -2)$ and the plane $7x - 4y - 4z + 79 = 0$ is 9 units.

29) Calculate the value of parameter p for which the distance between the point $A\,(4,\ 12,\ -4)$ and the plane $px - y + 2z + 40 = 0$ is 4 units.

30) Discuss the intersection of the following plane and line. Find out the intersection point if possible:

$$\pi:\ 5x - 3y - 3z + 4 = 0, \qquad r: \begin{cases} 2x - y - 5z + 5 = 0 \\ 8x - 5y - z = 0 \end{cases}$$

31) Discuss the intersection of the following plane and line. Find out the intersection point if possible:

$$\pi:\ x + y - 2z - 7 = 0, \qquad r: \begin{cases} 2x + y - 3z - 5 = 0 \\ 3x + y - 4z - 3 = 0 \end{cases}$$

32) Discuss the intersection of the following plane and line. Find out the intersection point if possible:

$$\pi:\ x + 3y - 4z - 4 = 0, \qquad r: \begin{cases} 4x - y - 4z + 9 = 0 \\ 4x + 5z + 17 = 0 \end{cases}$$

33) Discuss the intersection of the following plane and line according to the values of parameter m. Find out the intersection point (if possible) when parameter is $m = 7$.

$$\pi:\ -x + my + 6z = -19, \qquad r: \begin{cases} x + 4y + 2z = 10 \\ x + 8y + 6z = -2 \end{cases}$$

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34) Discuss the intersection of the following plane and line according to the values of parameter n and find the intersection point if possible.

$$\pi: \; x - 3y - 4z = -3, \qquad r: \begin{cases} 2x - 5y + nz = -3 \\ x - 2y - z = 0 \end{cases}$$

35) Discuss the relative position of the following lines. Find out the intersection point if possible.

$$r_1: \begin{cases} x = -13 - 5\mu \\ y = 6 + 2\mu \\ z = 3 + 2\mu \end{cases}, \qquad r_2: \begin{cases} x = -10 - \lambda \\ y = -2\lambda \\ z = 11 + 5\lambda \end{cases}$$

36) Discuss the relative position of the following lines. Find out the intersection point if possible.

$$r_1: \frac{x+1}{5} = \frac{y-6}{1} = \frac{z-4}{-1}, \qquad r_2: \frac{x}{-4} = \frac{y-9}{-6} = \frac{z+3}{0}$$

37) Discuss the relative position of the following lines. Find out the intersection point if possible.

$$r_1: \frac{x-8}{-3} = \frac{y-12}{6} = \frac{z+6}{-2}, \qquad r_2: \begin{cases} x = 5 + 6\lambda \\ y = 18 - 12\lambda \\ z = -8 + 4\lambda \end{cases}$$

38) Find out the relative position of the following lines according to the values of parameter r.

$$r_1: \; (x, y, z) = (-3, -6, 12) + \lambda\,(4, 8, -1), \qquad r_2: \frac{x-5}{-12} = \frac{y-10}{r} = \frac{z-10}{3}$$

39) Discuss the relative position of the following lines according to the values of parameter q and calculate the intersection point if possible.

$$r_1: \begin{cases} x = 13 + 8\lambda \\ y = -24 - 8\lambda \\ z = 13 + 3\lambda \end{cases}, \qquad r_2: \frac{x+3}{0} = \frac{y+22}{-7} = \frac{z-q}{-3}$$

40) Find out the relative position of the following lines according to the values of parameter r.

$$r_1: \begin{cases} x = -9 - 5\lambda \\ y = 2 + 5\lambda \\ z = 1 - \lambda \end{cases}, \qquad r_2: \frac{x-10}{-10} = \frac{y-11}{10} = \frac{z+7}{r}$$

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Answers:

1) 418 u^3.

2) **a)** 33; 29 **b)** −188 **c)** 101° 19' 45.41"

3) **a)** 97° 36' 44.39" **b)** 50° 9' 13.41"

4) **a)** $\left(\frac{1}{3}, \frac{-14}{39}, \frac{-34}{39}\right)$ **b)** $\left(\frac{2}{3}, \frac{11}{15}, \frac{2}{15}\right)$

5) $x = 1, \quad y = 1, \quad z = 8.$

6) $p = \frac{152}{15}; \quad k = 16$

7) $\left(\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}\right)$

8) **a)** $r = \pm 32$ **b)** $r = \frac{-5}{3}$ **c)** $r = \frac{1971}{5}.$

9) $k \neq \frac{456}{233}.$

10) $8x - 3y - 11z - 63 = 0$

11) $6x - 5y - 3z - 9 = 0$

12) $3x + 8y + 7z + 21 = 0$

13) $20x - 6y - 9z + 31 = 0$

14) $20x - 4y - 15z - 60 = 0$

15) $11x - 4y - 4z + 56 = 0$

16) $y - z = 0$

17) $11x - 6y - 7z + 13 = 0$

18) $x - z - 9 = 0$

19) $x + y - z - 4 = 0$

20) $\frac{x+3}{-4} = \frac{y+5}{9} = \frac{z-12}{9}$

21) $\frac{x-10}{-1} = \frac{y+7}{2} = \frac{z-11}{4}$

22) $\frac{x-8}{10} = \frac{y+3}{7} = \frac{z}{1}$

23) $\frac{x-8}{6} = \frac{y-1}{1} = \frac{z+1}{9}$

24) $\frac{x+8}{10} = \frac{y+3}{-19} = \frac{z+3}{13}$

25) $\frac{x-5}{2} = \frac{y+9}{-3} = \frac{z+7}{-1}$

26) $\frac{x}{-3} = \frac{y+7}{-1} = \frac{z+6}{-4}$

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28) $r_1 = 12, \quad r_2 = \frac{105}{2}$

29) $p = -2$

30) The line is parallel to the plane

31) The line lies in the plane

32) They intersect at a point $P\,(-3,\ 1,\ -1)$

33) $m = 4 \rightarrow$ The line is parallel to the plane. $m \neq 4 \rightarrow$ They intersect at a point.
$m = 7 \rightarrow$ They intersect at a point $P\,(6,\ 5,\ -8)$

34) $n = -5 \rightarrow$ The line lies in the plane.
$n \neq -5 \rightarrow$ They intersect at a point $P\,(6,\ 3,\ 0)$.

35) Intersect at a point, $P\,(-8,\ 4,\ 1)$

36) Don't intersect and not parallel

37) Overlap

38) $r \neq -24 \rightarrow$ Intersect at a point.
$r = -24 \rightarrow$ Overlap.

39) $q \neq 1 \rightarrow$ Don't intersect and not parallel.
$q = 1 \rightarrow$ Intersect at a point, $P\,(-3,\ -8,\ 7)$.

40) $r \neq -2 \rightarrow$ Don't intersect and not parallel.
$r = -2 \rightarrow$ Parallel.