Problems of Cayley–Hamilton Theorem

1) Calculate A^2 and A^3 as a linear combination of the matrix $A = \begin{pmatrix} 8 & 0 \\ 5 & 1 \end{pmatrix}$ and the identity matrix using the Cayley–Hamilton Theorem.

2) Calculate A^2 and A^3 as a linear combination of the matrix $A = \begin{pmatrix} -5 & 5 \\ 0 & 5 \end{pmatrix}$ and the identity matrix using the Cayley–Hamilton Theorem.

3) Calculate A^{-1} and A^2 as a linear combination of the matrix $A = \begin{pmatrix} 2 & 6 \\ 2 & 1 \end{pmatrix}$ and the identity matrix using the Cayley–Hamilton Theorem.

4) Find A^{-1} and A^4 as a linear combination of the matrix $A = \begin{pmatrix} -3 & 8 \\ -1 & -7 \end{pmatrix}$ and the identity matrix using the Cayley–Hamilton Theorem.

Problems of Cayley–Hamilton Theorem

Answers:

1) $A^2 = 9A - 8I = \begin{pmatrix} 64 & 0 \\ 45 & 1 \end{pmatrix}, \quad A^3 = 73A - 72I = \begin{pmatrix} 512 & 0 \\ 365 & 1 \end{pmatrix}$

2) $A^2 = 25I = \begin{pmatrix} 25 & 0 \\ 0 & 25 \end{pmatrix}, \quad A^3 = 25A = \begin{pmatrix} -125 & 125 \\ 0 & 125 \end{pmatrix}$

3) $A^{-1} = \dfrac{A - 3I}{10} = \dfrac{1}{10}\begin{pmatrix} -1 & 6 \\ 2 & -2 \end{pmatrix}, \quad A^2 = 3A + 10I = \begin{pmatrix} 16 & 18 \\ 6 & 13 \end{pmatrix}$

4) $A^{-1} = \dfrac{-A - 10I}{29} = \dfrac{1}{29}\begin{pmatrix} -7 & -8 \\ 1 & -3 \end{pmatrix}, \quad A^4 = -420A - 2059I = \begin{pmatrix} -799 & -3360 \\ 420 & 881 \end{pmatrix}$