## Problems of Matrices: Change of basis matrix

**1)** Find the coordinates of the vector $\mathbf{w} = \begin{pmatrix} -14 \\ -8 \end{pmatrix}$ relative to the basis $\mathbf{B} = \left\{ \begin{pmatrix} 1 \\ 7 \end{pmatrix}, \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$.

**2)** Find the coordinates of the vector $\mathbf{w} = \begin{pmatrix} 20 \\ 18 \end{pmatrix}$ relative to the basis $\mathbf{B} = \left\{ \begin{pmatrix} -3 \\ -4 \end{pmatrix}, \begin{pmatrix} 7 \\ 5 \end{pmatrix} \right\}$.

**3)** Find the coordinates of the vector $\mathbf{w} = \begin{pmatrix} -3 \\ -15 \\ 18 \end{pmatrix}$ relative to the basis $\mathbf{B} = \left\{ \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} -2 \\ 3 \\ -6 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \right\}$.

**4)** Find the coordinates of the vector $\mathbf{w} = \begin{pmatrix} -12 \\ 12 \\ 0 \end{pmatrix}$ relative to the basis $\mathbf{B} = \left\{ \begin{pmatrix} -3 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 8 \end{pmatrix} \right\}$.

**5)** Let **B** and **C** two bases for $\Re^2$:

$$\mathbf{B} = \left\{ \begin{pmatrix} -2 \\ -5 \end{pmatrix}, \begin{pmatrix} -4 \\ -3 \end{pmatrix} \right\}, \quad \mathbf{C} = \left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right\}.$$

Calculate the change of basis matrix from **B** to **C**.

**6)** Let **B** and **C** two bases for $\Re^2$:

$$\mathbf{B} = \left\{ \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}, \quad \mathbf{C} = \left\{ \begin{pmatrix} -1 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}.$$

**a)** Calculate the change of basis matrix from **B** to **C**.

Let the vector **v** with coordinates $\mathbf{v_B} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$ relative to the basis **B**. Find the coordinates of the vector **v** relative to the standard basis and to the basis **C**.

**7)** Let **B** and **C** two bases for $\Re^3$:

$$\mathbf{B} = \left\{ \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -5 \\ -1 \\ 0 \end{pmatrix} \right\}, \quad \mathbf{C} = \left\{ \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \right\}.$$

**a)** Calculate the change of basis matrix from **B** to **C**.

Let the vector **v** with coordinates $\mathbf{v_B} = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ relative to the basis **B**. Find the coordinates of the vector **v** relative to the standard basis and to the basis **C**.

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**8)** Let **B** and **C** two bases for $\Re^3$:

$$\mathbf{B} = \left\{ \begin{pmatrix} -5 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 0 \\ -5 \end{pmatrix}, \begin{pmatrix} -5 \\ -5 \\ -3 \end{pmatrix} \right\}, \quad \mathbf{C} = \left\{ \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}.$$

Calculate the change of basis matrix from **B** to **C**.

**Answers:**

**1)** $\mathbf{w_B} = \begin{pmatrix} -2 \\ 6 \end{pmatrix}$

**2)** $\mathbf{w_B} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

**3)** $\mathbf{w_B} = \begin{pmatrix} -3 \\ -3 \\ 6 \end{pmatrix}$

**4)** $\mathbf{w_B} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}$

**5)** $P_{\mathbf{BC}} = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}.$

**6)** **a)** $P_{\mathbf{BC}} = \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix}$, **b)** $\mathbf{v} = \begin{pmatrix} -4 \\ -7 \end{pmatrix}$, $\mathbf{v_C} = P_{\mathbf{BC}} \cdot \mathbf{v_B} \rightarrow \mathbf{v_C} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$

**7)** **a)** $P_{\mathbf{BC}} = \begin{pmatrix} -1 & 1 & 2 \\ 0 & -1 & -1 \\ 0 & 1 & -2 \end{pmatrix}$, **b)** $\mathbf{v} = \begin{pmatrix} -12 \\ -12 \\ -6 \end{pmatrix}$, $\mathbf{v_C} = P_{\mathbf{BC}} \cdot \mathbf{v_B} \rightarrow \mathbf{v_C} = \begin{pmatrix} 0 \\ 0 \\ -6 \end{pmatrix}.$

**8)** $P_{\mathbf{BC}} = \begin{pmatrix} -1 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -2 & -1 \end{pmatrix}.$