Problems of Complex numbers

1) Calculate and simplify the following expressions:

a) $\left(\frac{-5}{4} + \frac{14}{5}\,i\right) + \left(10 - \frac{10}{9}\,i\right)$

b) $(8 + 27\,i) - (10 + 11\,i)$

c) $\left(\frac{1}{2} + 3\,i\right) - \left(-9 - \frac{5}{3}\,i\right)$

d) $\left(\frac{-5}{3} + 8\,i\right) + \left(\frac{24}{7} + \frac{3}{10}\,i\right)$

e) $\left(\frac{1}{5} + 6\,i\right) + (23 + 13\,i)$

f) $\left(\frac{15}{11} + \frac{1}{2}\,i\right) - (-1 + 25\,i)$

2) Calculate and simplify the following expressions:

a) $\frac{-10 + 26\,i}{3 + 7\,i}$

b) $\frac{5 + 25\,i}{9 + 8\,i}$

c) $\frac{14 + 6\,i}{10 - 4\,i}$

d) $\frac{3 + 8\,i}{-8 + 9\,i}$

e) $\frac{18 + 5\,i}{10 + 4\,i}$

f) $(8 + 8\,i) \times (7 + 4\,i)$

3) Find out the polar form of the following complex numbers:

a) $7 + 9\,i$

b) $16\,i$

c) $19 + 23\,i$

d) $21 + 8\,i$

e) $25 + 24\,i$

f) $21 + 3\,i$

4) Find out the cartesian form of the following complex numbers:

a) $26_{1.96559\text{ rad}}$

b) $4_{\frac{3\pi}{2}\text{ rad}}$

c) $2_{270°}$

d) $\sqrt{1170}_{52.125°}$

e) $\sqrt{221}_{0.832981\text{ rad}}$

f) $\sqrt{442}_{87.2737°}$

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5) Calculate the following operations of complex numbers. Give the results in cartesian and polar forms.

a) $\dfrac{45_{323.13°}}{15_{53.1301°}}$

b) $\left(\sqrt{80}_{116.565°}\right) \times \left(\sqrt{20}_{243.435°}\right)$

c) $\dfrac{\sqrt{5200}_{109.44°}}{\sqrt{80}_{26.5651°}}$

d) $\left(\sqrt{185}_{36.0274°}\right) \times \left(\sqrt{122}_{95.1944°}\right)$

e) $\dfrac{\sqrt{16400}_{104.47°}}{10_{53.1301°}}$

f) $\left(\sqrt{10}_{251.565°}\right) \times \left(\sqrt{17}_{255.964°}\right)$

6) Find out the following calculations. Give the results in cartesian and polar forms.

a) $(5 + i)^2$

b) $\left(4_{90°}\right)^2$

c) $(-2 - 3\,i)^3$

d) $\left(2_{270°}\right)^4$

e) $(-2 - 4\,i)^2$

f) $\left(\sqrt{13}_{303.69°}\right)^3$

7) Calculate the following roots of complex numbers. Give the results in polar form.

a) $\sqrt[3]{27_{90°}}$

b) $\sqrt[3]{26 + 18\,i}$

c) $\sqrt{32_{270°}}$

d) $\sqrt{9 + 40\,i}$

e) $\sqrt{45_{233.13°}}$

f) $\sqrt{-40 - 42\,i}$

8) Let the complex numbers $z = c - 4i$ and $w = -8 - 4\,i$, calculate the value of parameter c for which the multiplication of these complex numbers gives a real number.

9) Solve the following complex equations:

a) $z^2 = -36$

b) $z^2 - 4z = -29$

c) $z^4 + 10000 = 0$

d) $z^2 - 4 - 4\sqrt{3}\,i = 0$

e) $z^4 - 6561 = 0$

f) $z^2 + 144 = 0$

10) Find all complex numbers satisfying the following equations and plot the results on an Argand diagram (complex plane):

a) $z^3 + 5832\,i = 0$

b) $z^2 + 6z + 18 = 0$

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Answers:

1) **a)** $\frac{35}{4} + \frac{76}{45}\,i$ **b)** $-2 + 16\,i$ **c)** $\frac{19}{2} + \frac{14}{3}\,i$ **d)** $\frac{37}{21} + \frac{83}{10}\,i$

e) $\frac{116}{5} + 19\,i$ **f)** $\frac{26}{11} - \frac{49}{2}\,i$

2) **a)** $\frac{76}{29} + \frac{74}{29}\,i$ **b)** $\frac{49}{29} + \frac{37}{29}\,i$ **c)** $1 + i$ **d)** $\frac{48}{145} - \frac{91}{145}\,i$

e) $\frac{50}{29} - \frac{11}{58}\,i$ **f)** $24 + 88\,i$

3) **a)** $\sqrt{130}_{52.125^\circ}$ **b)** $16_{\frac{\pi}{2}\text{ rad}}$ **c)** $\sqrt{890}_{50.4403^\circ}$

d) $\sqrt{505}_{20.8545^\circ}$ **e)** $\sqrt{1201}_{0.764993\text{ rad}}$ **f)** $\sqrt{450}_{0.141897\text{ rad}}$

4) **a)** $-10 + 24\,i$ **b)** $-4\,i$ **c)** $-2\,i$

d) $21 + 27\,i$ **e)** $10 + 11\,i$ **f)** $1 + 21\,i$

5) **a)** $3_{270^\circ} = -3\,i$ **b)** $40_{0^\circ} = 40$

c) $\sqrt{65}_{82.875^\circ} = 1 + 8\,i$ **d)** $\sqrt{22570}_{131.222^\circ} = -99 + 113\,i$

e) $\sqrt{164}_{51.3402^\circ} = 8 + 10\,i$ **f)** $\sqrt{170}_{147.529^\circ} = -11 + 7\,i$

6) **a)** $26_{22.6199^\circ} = 24 + 10\,i$ **b)** $16_{180^\circ} = -16$

c) $\sqrt{2197}_{348.93^\circ} = 46 - 9\,i$ **d)** $16_{0^\circ} = 16$

e) $20_{126.87^\circ} = -12 + 16\,i$ **f)** $\sqrt{2197}_{191.07^\circ} = -46 - 9\,i$

7) **a)** 3_{270°, 3_{30°, 3_{150°

b) $\sqrt{10}_{251.565^\circ}$, $\sqrt{10}_{11.5651^\circ}$, $\sqrt{10}_{131.565^\circ}$

c) $\sqrt{32}_{315^\circ}$, $\sqrt{32}_{135^\circ}$

d) $\sqrt{41}_{38.6598^\circ}$, $\sqrt{41}_{218.66^\circ}$

e) $\sqrt{45}_{116.565^\circ}$, $\sqrt{45}_{296.565^\circ}$

f) $\sqrt{58}_{113.199^\circ}$, $\sqrt{58}_{293.199^\circ}$

8) $c = 8$

9) **a)** $z_1 = 6\,i,\ \ z_2 = -6\,i$

b) $z_1 = 2 + 5\,i,\ \ z_2 = 2 - 5\,i$

c) $z_1 = \sqrt{50} + \sqrt{50}\,i,\ \ z_2 = -\sqrt{50} + \sqrt{50}\,i,\ \ z_3 = -\sqrt{50} - \sqrt{50}\,i,\ \ z_4 = \sqrt{50} - \sqrt{50}\,i$

d) $z_1 = \sqrt{6} + \sqrt{2}\,i,\ \ z_2 = -\sqrt{6} - \sqrt{2}\,i$

e) $z_1 = 9,\ \ z_2 = 9\,i,\ \ z_3 = -9,\ \ z_4 = -9\,i$

f) $z_1 = 12\,i,\ \ z_2 = -12\,i$

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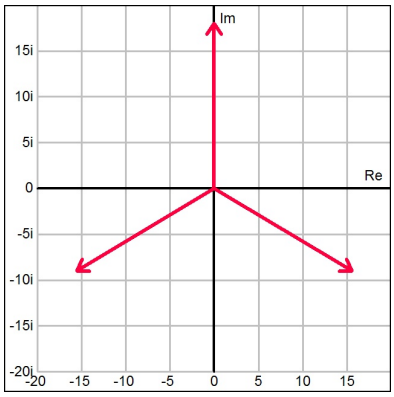
10) **a)** $z_1 = 18\,i,\quad z_2 = -\sqrt{243} - 9\,i,\quad z_3 = \sqrt{243} - 9\,i$



b) $z_1 = -3 + 3\,i,\quad z_2 = -3 - 3\,i$