

Problems of Concavity and inflection points of functions

1) Calculate the inflection points of the following functions:

a) $f(x) = x^3 + 12x^2 - 7$

b) $f(x) = \frac{-7}{4x^2 + 108}$

c) $f(x) = (15x + 10) \cdot e^{3x + 4}$

d) $f(x) = \ln(4x^2 + 36)$

e) $f(x) = -3x^3 + 18x^2$

f) $f(x) = \frac{-8}{-6x^2 - 162}$

2) Find out the intervals of concavity (concave up/down) and the inflection points of the following function:

$$f(x) = (x + 1)^2(x - 4)$$

3) Find out the intervals of concavity (concave up/down) and the inflection points of the following function:

$$f(x) = \frac{-8}{x^2 + 4}$$

4) Find out the intervals of concavity (concave up/down) and the inflection points of the following function:

$$f(x) = \frac{4x}{x^2 + 5}$$

Problems of Concavity and inflection points of functions

Answers:

1) a) $(-4, 121)$

b) $\left(-3, \frac{-7}{144}\right), \left(3, \frac{-7}{144}\right)$

c) $\left(\frac{-4}{3}, -10\right)$

d) $(-3, \ln 72), (3, \ln 72)$

e) $(2, 48)$

f) $\left(-3, \frac{1}{27}\right), \left(3, \frac{1}{27}\right)$

2) Concave downwards on $\left(-\infty, \frac{2}{3}\right)$. Concave upwards on $\left(\frac{2}{3}, +\infty\right)$.

Inflection point $\left(\frac{2}{3}, \frac{-250}{27}\right)$.

3) Concave downwards on $\left(-\infty, -\sqrt{\frac{4}{3}}\right) \cup \left(\sqrt{\frac{4}{3}}, +\infty\right)$.

Concave upwards on $\left(-\sqrt{\frac{4}{3}}, \sqrt{\frac{4}{3}}\right)$.

Inflection points: $\left(-\sqrt{\frac{4}{3}}, \frac{-3}{2}\right), \left(\sqrt{\frac{4}{3}}, \frac{-3}{2}\right)$.

4) Concave downwards on $\left(-\infty, -\sqrt{15}\right) \cup \left(0, \sqrt{15}\right)$.

Concave upwards on $\left(-\sqrt{15}, 0\right) \cup \left(\sqrt{15}, +\infty\right)$.

Inflection points: $\left(-\sqrt{15}, -\frac{\sqrt{15}}{5}\right), (0, 0), \left(\sqrt{15}, \frac{\sqrt{15}}{5}\right)$.