

Problems of Definite integrals

- 1) Sketch the graph of the curve $y = \frac{-4}{x+5}$ and find the area between this curve, the X axis, and the vertical lines $x = -4$ and $x = 0$.
- 2) Sketch the graph of the curve $y = x^2 - 7x + 6$ and find the area between this curve, the X axis, and the vertical lines $x = 2$ and $x = 4$.
- 3) Sketch the graph and find the area between the curve $y = x^2 - x - 2$ and the X axis.
- 4) Sketch the graph of the curve $y = (4x - 4) \ln(x - 3)$ and find the area between this curve, the X axis, and the vertical lines $x = 6$ and $x = 8$.
- 5) Sketch the graph and find the area between the curve $y = x^3 - 8x$ and the line $y = x$.
- 6) Sketch the graph and find the area between the hyperbola $y = \frac{6}{x+5}$ and the line that cuts this hyperbola at the points $x = -4$ and $x = -2$.
- 7) The area between the parabola $y = kx - x^2$ and the X axis is $\frac{125}{48}$. Find out the value of the parameter k where $k > 0$.
- 8) The curve $y = kx^2 - x^3$ cuts the X axis at the points $(0, 0)$ and $(k, 0)$. Find out the value of the parameter k if the area between the curve and the X axis is 108 and $k > 0$.
- 9) We have the derivative of a function $f'(x) = \frac{1}{x-3}$, find out the expression for the function $f(x)$ whose graph passes through the point $(4, -2)$.
- 10) We know the derivative $f'(x) = \frac{7}{(x+1)^2}$ of a function. Calculate the expression for this function $f(x)$ if its graph passes through the point $(0, 5)$.
- 11) The first derivative of a function is $f'(x) = 8x + e^{x+6}$. Find out the expression for the function $f(x)$ whose graph passes through the point $(-6, 157)$.
- 12) Consider the second derivative $f''(x) = 18x$. If the graph of the function passes through the point $(0, 19)$ and at the point $x = 1$ the value of its first derivative is 0, find out the expression of the function $f(x)$.

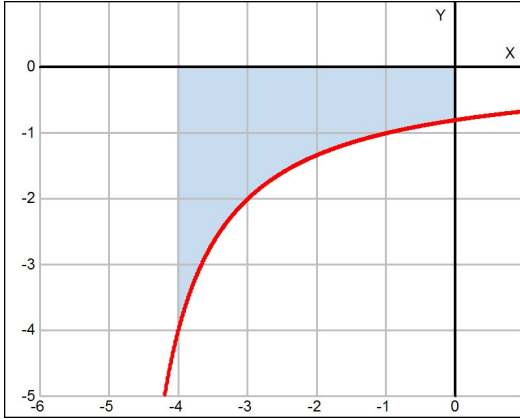
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- 13)** We know the second derivative $f''(x) = 2 + e^x$ of a function. If the graph of this function passes through the point $(0, 19)$ and the value of its first derivative is 10 at $x = 0$, determine the expression of $f(x)$.
- 14)** We have the second derivative of a function $f''(x) = -4 - \frac{1}{x^2}$. The graph of the function passes through the point $(1, -15)$ and the value of the first derivative is 5 when $x = 1$, calculate the expression of $f(x)$.
- 15)** Find out the volume generated when the curve $y = \sqrt{\cos x}$ is rotated about the x -axis between the planes with equations $x = 0$ and $x = \pi/2$.
- 16)** Consider the region R bounded by the circle $x^2 + y^2 = 25$. Calculate the volume generated when R is rotated about the x -axis.
- 17)** Consider the region R bounded by the ellipse $\frac{x^2}{196} + \frac{y^2}{49} = 1$. Find out the volume generated when R is rotated about the x -axis.
- 18)** Calculate the volume generated when the parabola $y^2 = 5x$ is rotated about the x -axis between the planes with equations $x = 0$ and $x = 3$.

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Answers:

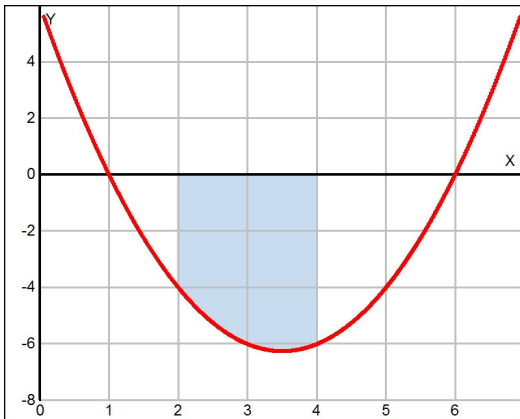
- 1) There is no X-intercept



$$A = - \int_{-4}^0 \frac{-4}{x+5} dx$$

$$A = 4 \ln 5 \cong 6.4378 \text{ u}^2.$$

- 2) X-intercepts (1, 0), (6, 0)

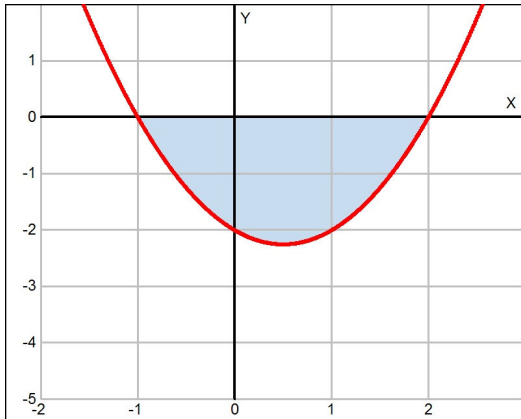


$$A = \int_1^6 (-x^2 + 7x - 6) dx$$

$$\frac{34}{3} \text{ u}^2.$$

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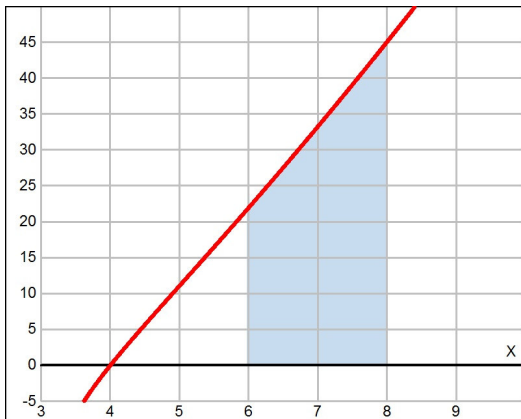
3) X-intercepts $(-1, 0)$, $(2, 0)$



$$A = \int_{-1}^2 (-x^2 + x + 2) dx$$

$$A = \frac{9}{2} u^2.$$

4) X-intercept $(4, 0)$

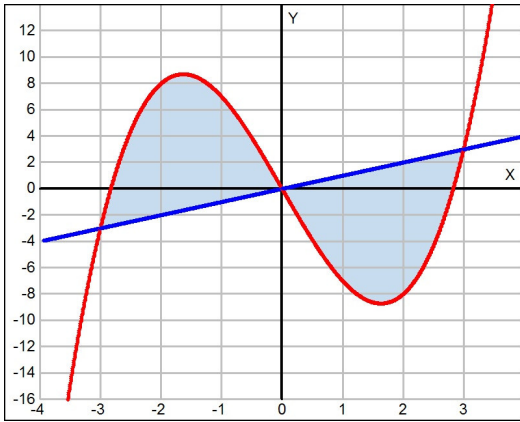


$$A = \int_6^8 (4x - 4) \ln(x - 3) dx$$

$$A = -32 + 90 \ln 5 - 42 \ln 3 \cong 66.7077 u^2.$$

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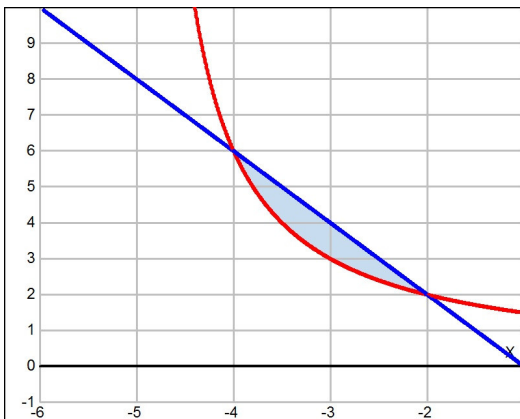
- 5) Intersection points $(-3, -3)$, $(0, 0)$, $(3, 3)$



$$A = 2 \int_0^3 [x - (x^3 - 8x)] dx = \int_0^3 (-2x^3 + 18x) dx$$

$$A = \frac{81}{2} u^2.$$

- 6) Intersection points $(-4, 6)$, $(-2, 2)$. Secant line: $y = -2x - 2$



$$A = \int_{-4}^{-2} \left(-2x - 2 - \frac{6}{x+5} \right) dx$$

$$A = 8 - 6 \ln 3 u^2 \cong 1.40833 u^2.$$

7) $k = \frac{5}{2}.$

8) $k = 6.$

9) $f(x) = \ln(x - 3) - 2.$

10) $f(x) = \frac{-7}{x+1} + 12.$

11) $f(x) = 4x^2 + e^{x+6} + 12.$

12) $f(x) = 3x^3 - 9x + 19.$

13) $f(x) = x^2 + 9x + 18 + e^x.$

14) $f(x) = -2x^2 + 8x - 21 + \ln x.$

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16) $\frac{500}{3}\pi u^3$

17) $\frac{2744}{3}\pi u^3$

18) $\frac{45}{2}\pi u^3$